DOCUMENT RESUME

ED 106 114 SE 019 017

TITLE	Basic Mathematics, Programmed Text. Edition 9. Engineer Subcourse 120-9.
INSTITUTION	Army Engineer School, Fort Belvoir, Va.
PUB DATE	Apr 74
NOTE	282p.
EDRS PRICE	MF-$0.76 HC-$14.59 PLUS POSTAGE
DESCRIPTORS	*Basic Skills; *Engineering Education; Independent Study; Instructional Materials; *Mathematics Education; *Post Secondary Education; *Programed Instruction; Textbooks

ABSTRACT

This course is designed to teach the student to perform basic mathematics operations correctly. It consists of seven lessons and an examination as follows: Introduction to Arithmetic and Whole Numbers, Common Fractions, Decimals, Unit Conversion, Ratios and Proportions, Percentage, Powers and Roots, and the examination. The course uses the technique of programed instruction. (Author/KM)

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ARMY CORRESPONDENCE COURSE • ENGINEER SUBCOURSE 120-9

ED106114

PROGRAMMED TEXT

BASIC MATHEMATICS

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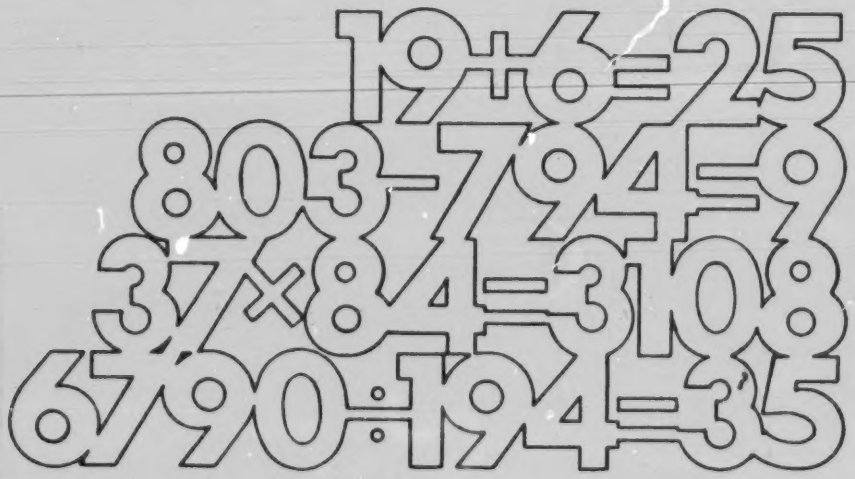


SE 019 017

United States Army Engineer School • EDITION 9 (APRIL 1974)

INTRODUCTION

Contents

This subcourse is designed to teach you to correctly perform the basic mathematics operations. Such knowledge will enable you to better utilize techniques of mathematics and will increase your capability of assuming greater responsibilities.

The subcourse consists of seven lessons and an examination as follows:

Lesson 1. Introduction to Arithmetic and Whole Numbers
2. Common Fractions
3. Decimals
4. Unit Conversion
5. Ratios and Proportions
6. Percentage
7. Powers and Roots

Examination

Twelve credit hours are allowed for the subcourse.

Presentation

This subcourse uses the technique of programmed instruction. Essentially, programmed instruction is the presentation of ideas in a logically sound learning sequence. It presents material in small bits; it provides cues to the right answer; it often repeats key ideas in a different form or content; and it reinforces immediately the student's correct response.

Description of Programmed Text

At the beginning of each lesson, you will find a list of Objectives that will tell you exactly what you must know. The Program will TEACH you exactly what the Objectives state. Read them carefully.

Two types (techniques) of Programming will be used: "linear" and "branching".

The Linear Program has small bits of information in "frames." Each frame will have a question that you must answer concerning what you have been taught in that or previous frames. Fill in the blanks in pencil with the appropriate word(s) or numerical quantities or solve the problems in the space provided. Your writing down the correct answer is an important aspect of the teaching technique. Turn to the next frame and check your written response against the correct answer that appears just above the frame number. If your response is correct, proceed with that frame. If your response is incorrect, restudy the frame answered incorrectly, erase your incorrect response and write the correct answer in the blank(s). It may seem quite easy to answer the questions at times, but study each question carefully and do not try to guess the answer. The Program has been designed so that it will teach easily and quickly. If you run across information that you are familiar with, DO NOT skip it. Use it as review material.

Example of LINEAR Programming:

FRAME 1-1

The average human with blond hair has about 120,000 hairs on his head. If you are blond, you have about_____hairs on your head.

(120,000) (1-1)

FRAME 1-2

More information will be taught in this frame and another question asked. After completing this frame, you will proceed to Frame 1-3 and so forth, through each frame of the lesson.

The Branching Program also has bits of information in frames, but you will be given a choice of answers. Each answer tells you to go to a certain frame. If the answer you select is correct, the frame you have turned to will tell you so and teach more information. If you were wrong, the frame will tell you that, too. It may reteach or send you to some other frame. The directions must be followed very closely in a Branching Program.

Example of Branch Programming:

FRAME 1-25

MOST humans have many hairs on their heads. The number of hairs varies with the color of it. Blonds have been found to have the most, while redheads have the least. Brunettes fall in between. The range is 80,000 to 120,000 hairs.

If you are an average brunette, about how many hairs do you have on your head?

If your answer is:	Go to Frame:
80,000	1-26 (p. 1-15)
100,000	1-27 (p. 1-20)
120,000	1-23 (p. 1-9)

Of course, your answer would be 100,000, so you should go to Frame 1-27.

FRAME 1-27

Right. You will be told here that your answer was correct and more material will be taught. Another question will be asked and 2 or more choices will be given.

If you had selected either of the other two answers, you would be told that they are wrong. You would be retaught or sent back.

Panels provide information in addition to that contained in the frames and will be referred to in the frames. The panels are located at the back of the programmed text at the end of the frames. When you are referred to a panel (for example, Panel 7-1, page 7-35), turn to the panel and study the information and then turn back to the frame on which you were working.

Exercise Booklet and Examination

Once you are satisfied that you understand the material in each lesson, go to the Exercise Booklet and solve the multiple choice exercises for that lesson. Indicate your answer choice on the answer sheet for that lesson. The answer sheets are bound in reverse order in the back of the Exercise Booklet.

You will not be limited as to the number of hours you may spend on the subcourse, any lesson, or the examination. For statistical purposes, you are required to enter in the proper space on the answer sheet the number of hours and minutes spent on each lesson, including the time for studying the programmed instruction text, solving the self-test problems, and solving the exercises.

The examination will be sent to you when you have successfully completed all the lessons.

YOU ARE NOW READY TO START LESSON ONE. READ EACH SENTENCE CAREFULLY; BE SURE YOU UNDERSTAND WHAT IS SAID BEFORE YOU TRY TO ANSWER THE QUESTION. TURN TO PAGE ONE OF LESSON ONE AND CAREFULLY READ THE LESSON OBJECTIVES.

LESSON 1

INTRODUCTION TO ARITHMETIC AND WHOLE NUMBERS

CREDIT HOURS ---------- 1

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Write, in his own words, the definition of a whole number.	1-1 (p.1-2)
2. Identify by name the "place" (position) of each digit in a given whole number.	1-5 (p.1-10)
3. Demonstrate the ability to read whole numbers by matching a list of numbers written as figures with a list of numbers written in word form.	1-9 (p.1-18)
4. Round off whole numbers.	1-12 (p.1-24)
5. Identify by name each number in given addition, subtraction, multiplication, and division problems.	1-19 (p.1-10)
6. Set up and solve given problems in addition, subtraction, multiplication, and division.	1-34 (p.1-13)
7. Solve and check given problems in addition, subtraction, multiplication, and division.	1-52 (p.1-21)

SET 1. DEFINITION OF A WHOLE NUMBER

Frames 1-1 through 1-14 are at the top of even numbered pages.

FRAME 1-1

A unit is a single quantity regarded as a whole in calculations. Whole numbers represent COMPLETE UNITS. For example, the whole number 2 would represent two complete units. A COMPLETE UNIT is represented by a _____ number.

(21,000) (1-14)

FRAME 1-15

Rounding off to the nearest FIVE hundred is somewhat different. If the number (to the hundreds place) is closer to 500 than it is to 000, then round to 500. For instance, 251 is closer to 500 than it is to 000, so 251 rounded to the nearest five hundred is _____

(dividend) (1-28)

FRAME 1-29

If the division doesn't come out evenly and there is a quantity left, that quantity is called the remainder. Label the parts of the problem below.

$$\begin{array}{r} 19 \\ 15\overline{)292} \\ \underline{15} \\ 142 \\ \underline{135} \\ 7 \end{array}$$

15-- _____

19-- _____

292-- _____

7-- _____

FRAME 1-43

You did not borrow correctly. $\begin{array}{r} 555 \\ -396 \\ \hline 259 \end{array}$ This is the way your problem appeared. After you borrow 1 from the hundreds place, it should change from 5 to a 4. Return to page 1-25, frame 1-40 and rework the problem.

(whole) (1-1)

FRAME 1-2

All whole numbers show or represent some "complete" unit. The definition of a whole number is:

A whole number represents a _____unit.

---

(500) (1-15)

FRAME 1-16

If the number is closer to 1,000 than it is to 500, it is rounded off to 1,000. 762 rounded to the nearest five hundred is _____.

(divisor; quotient; dividend; remainder)(1-29)

FRAME 1-30

If you have NOT missed any labels up to this point, go to page 1-11, frame 1-33 and work from there.

If you missed ANY, CHECK those labels that gave you the trouble AND complete frame 1-31 and 1-32 before continuing.

(2764 x 22 = 60808) (1-53)

FRAME 1-44

Right! Another thing to remember in multiplication is: ZERO times ANY number is ZERO. Example: 3,889,497 x 0 = 0.

Let's try division. The division problem 250 ÷ 25 = 10 says 250 divided by 25 equals 10. Set up in workable form, it would look like this:

$$25\overline{)250}\quad 10$$
$$\underline{25}$$
$$00$$

Notice that the number to the right of the division sign (25) is the divisor and goes outside the division block.

Work this problem: 450 ÷ 5 =

If your answer is:	Go to Frame:
90	1-54 (p. 1-25)
.01	1-50 (p. 1-17)

(complete) (1-2)

FRAME 1-?

A whole number represents a complete unit. This is the definition of a _____ _____.

(1,000) (1-16)

FRAME 1-17

Round off the following whole numbers:

415,001 to ten thousands_____

152,299 to hundred thousands_____ ___

12,636 to five hundreds_____

FRAME 1-31

Label the parts of the problems below.

a. $\begin{array}{r} 25 \\ \times\ 4 \\ \hline 100 \end{array}$

25-- _____

4-- _____

100-- _____

b. $\begin{array}{r} 311 \\ -\ 27 \\ \hline 284 \end{array}$

311-- _____

27-- _____

284-- _____

FRAME 1-45

Solve and check the following problems:

a. $10\overline{)352}$ check--

b. $\begin{array}{r} 12 \\ 16 \\ +\ 305 \end{array}$ check--

c. $\begin{array}{r} 15 \\ \times\ 3 \end{array}$ check--

d. $\begin{array}{r} 127 \\ -\ 39 \end{array}$ check--

ANSWERS ARE IN FRAME 1-56 (p. 1-29)

(whole number) (1-3)

FRAME 1-4

Write in your own words, the definition of a whole number.

_____ _____ _____ _____

_____ _____ _____

NOTE: If you need review of some part of the Program after you have already covered that section, you may turn back.

(420,000; 200,000; 12,500) (1-17)

FRAME 1-18

Round off each of the following whole numbers:

276 to five hundreds_____

549 to hundreds_____

11,637,520 to millions_____

132,399 to ten thousands_____

749,999 to hundred thousands_____

3,274 to tens_____

(a. multiplicand; multiplier; product;

b. minuend; subtrahend; remainder) (1-31)

FRAME 1-32

Label the parts of the problems below.

a. $\begin{array}{r} 10 \\ +\ 11 \\ \hline 21 \end{array}$

10-- _____

11-- _____

21-- _____

b. $20\overline{)155}\ 7$, $\begin{array}{r} 140 \\ \hline 15 \end{array}$

20-- _____

7-- _____

155-- _____

15-- _____

FRAME 1-46

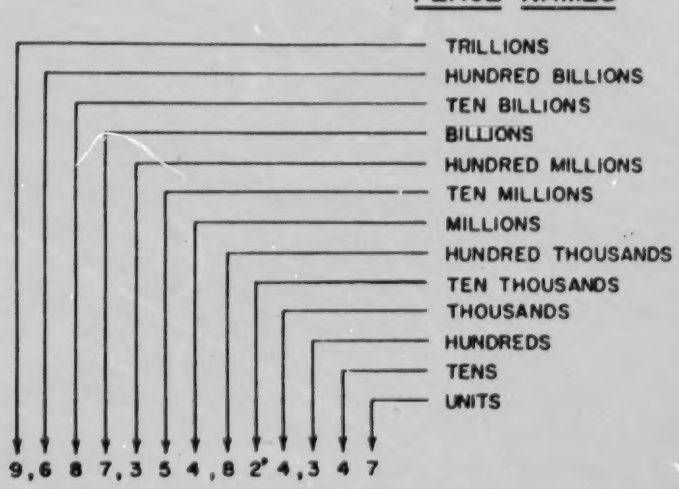
No! You must add the remainder to the subtrahend in order to obtain the minuend. This is the only way we know our subtraction is correct. If the sum of the check is not the same as the minuend in the problem, then you have made an error somewhere. Remember to ADD the remainder to the subtrahend to obtain the minuend. Go back to page 1-21, frame 1-52 and select the correct answer.

SET 2. NAME OF PLACES IN WHOLE NUMBER

(A whole number represents a complete unit) (1-4)

FRAME 1-5

Each digit in a whole number has a place value. The position of the digit in the number indicates its place value. The name of each place is given for the whole in the figure below.



The place to the far right is the_____place.

SET 5. NAMES OF NUMBERS IN ARITHMETIC OPERATION

(500; 500; 12,000,000; 130,000; 700,000; 3,270) (1-18)

FRAME 1-19

The answer determined from the addition of numbers is called the SUM. The numbers that are being added are called ADDENDS. Label the parts of the problem below:

$$\begin{array}{rr} & 4444 \\ + & 333 \\ \hline & 4777 \end{array}$$

(a. addend; addend; sum. b. divisor; quotient; dividend; remainder) (1-32)

FRAME 1-33

Label each part of each of the following addition, subtraction, multiplication, and division problems:

a. 289
488
+ 111
888

289-- _____

488-- _____

111-- _____

888-- _____

b. 397
- 44
353

397-- _____

44-- _____

353-- _____

c. 28
x 9
252

28-- _____

9-- _____

252-- _____

9 and 28 are also _____

d. $27\overline{)298}$ (11; 27; 28; 27; 1)

27-- _____

11-- _____

298-- _____

1-- _____

FRAME 1-47

Your check was not correct; you multiplied the remainder by the divisor when you should have multiplied the divisor by the quotient and then added the remainder. Your answer will be the dividend, if your original division was right. Go back to page 1-19, frame 1-51 and try the problem again.

(units) (1-5)

FRAME 1-6

The whole number 3,200 is to the thousands place. The two in the number is to the_____place.

(addend; addend; sum) (1-19)

FRAME 1-20

The parts in a subtraction problem are the minuend, subtrahend, and the remainder.

Example:
908 minuend
- 143 subtrahend
765 remainder

The answer or result is called the_____ .

SET 6. SET UP AND SOLVE ARITHMETIC PROBLEMS WITH WHOLE NUMBERS

(a.	addend	b.	minuend	c.	multiplicand	d.	divisor
	addend		subtrahend		multiplier		quotient
	addend		remainder		product		dividend
	sum				factors		remainder

If you missed any, review quickly and make your corrections.) (1-33)

FRAME 1-34

Any problem in addition, subtraction, multiplication, or division must be set up correctly in order to solve it. In ADDITION, you must put like units under like units.

Set up and work this addition problem. 3 + 297 + 48 =

If your answer is:	Go to Frame:
297 3 + 48 375	1-36 (p. 1-17)
297 48 + 3 448	1-38 (p. 1-21)
48 297 + 3 348	1-41 (p. 1-27)

FRAME 1-48

Wrong. You have placed the numbers directly below each other in the body of the multiplication. This is wrong. When your multiplication reaches the tens, hundreds, etc., place in your factors, the results will fall directly below the place. It will look like the other answer in frame 1-53 (p. 1-23), so return to frame 1-53 and work the problem correctly.

(hundreds) (1-6)

FRAME 1-7

A very large number such as 23,913,777 is out to the ten _____ place and the 1 in the number is in the _____ _____ place.

(remainder) (1-20)

FRAME 1-21

The larger number or the number you are subtracting from is called the _____ and the smaller number that is being subtracted from the larger number is called the _____.

FRAME 1-35

If you are reading this paragraph, you are not following directions. The preceding frame has given you directions to follow. These directions MUST be followed and you must be more careful when reading. Return to frame 1-34 and READ CAREFULLY.

$$\begin{array}{r} 455 \\ -\ 50 \\ \hline 405 \end{array} \quad \text{check} \quad \begin{array}{r} 405 \\ +\ 50 \\ \hline 455 \end{array} \quad \text{(1-52)}$$

FRAME 1-49

Very good. To check addition problems, invert the column and add. It is that simple.

Multiplication is checked by dividing the product by either the multiplicand or the multiplier. For example: $2 \times 3 = 6$ can be checked by dividing $2\overline{)\begin{array}{c}3\\6\end{array}}$ or $3\overline{)\begin{array}{c}2\\6\end{array}}$. Notice that the dividend is the product of the multiplication problem. Remember that you may use either one of the factors in the divisor spot, but the quotient must be the other factor.

Solve and check this problem. $4 \times 8 =$

$4 \times 8 = 32$ $\quad 8\overline{)32}$ or $4\overline{)32}$

For correct answer go to frame 1-51 (p. 1-19).

(millions; ten thousands) (1-7)

FRAME 1-8

Label the place of each digit in the whole number below.

(805,732,567)

8 _____ _____

0 _____ _____

5,_____

7 _____ _____

3 _____ _____

2, _____

5 _____

6 _____

7 _____

(minuend; subtrahend) (1-21)

FRAME 1-22

Label the parts of the problem below.

3,002 _____
- 3,001 _____
1 _____

FRAME 1-36

Your addition is fine, but remember that you must put like units under like units. It makes no difference where you put the large numbers in the sequence but units must be under units, tens under tens, etc. The addition problem, 25 + 3920 would be set up like this:

$$\begin{array}{r} 3920 \\ +\ 25 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 25 \\ +\ 3920 \\ \hline \end{array}$$

Set up and solve this problem: 27 + 598 + 20047 =

If your answer is:	Go to Frame:
$\begin{array}{r} 20047 \\ 27 \\ +\ 598 \\ \hline 20672 \end{array}$	1-42 (p. 1-29)
$\begin{array}{r} 598 \\ 20047 \\ +\ 27 \\ \hline 20572 \end{array}$	1-39 (p. 1-23)

FRAME 1-50

The only possible way to get the answer you have is to set up the problem incorrectly. 450 ÷ 5 is set up like this: $5\overline{)450}$. It states 450 divided by 5. The number to the right of the division sign always goes outside the division block. Return to page 1-5, frame 1-44 and work the problem correctly. Then select the correct answer and go to the indicated page.

SET 3. READING WHOLE NUMBERS

(hundred millions; ten millions; millions; hundred thousands; ten thousands; thousands; hundred; tens; units) (1-8)

FRAME 1-9

A whole number is read from the left. For example, the whole number 524 is read five hundred twenty-four. One thousand twenty-six would be shown as a whole number as _____ (number).

_____

(minuend; subtrahend; remainder) (1-22)

FRAME 1-23

Label the parts of these problems.

a.
$$\begin{array}{r} 678 \\ +\ 28 \\ \hline 706 \end{array}$$
_____
_____
_____

b.
$$\begin{array}{r} 67 \\ -\ 7 \\ \hline 60 \end{array}$$
_____
_____
_____

FRAME 1-37

The smaller number must go on the bottom. Subtraction that is accurate is not possible the way it was set up. Always put the smaller number below the larger. Go back to page 1-27, frame 1-41 and select the correct answer.

$$\left[\begin{array}{r} 4 \\ 8\overline{)32} \\ 32 \\ \hline 0 \end{array} \;;\; \begin{array}{r} 8 \\ 4\overline{)32} \\ 32 \\ \hline 0 \end{array}\right] \quad (1\text{-}49)$$

FRAME 1-51

Right. All that is left to check is division. This is done by multiplying the divisor by the quotient and adding the remainder (if any). For example:

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ 15 \\ \hline 2 \end{array}$$

check

$$\begin{array}{r} 5 \text{ quotient} \\ \times\ 3 \text{ divisor} \\ \hline 15 \\ +\ 2 \text{ remainder} \\ \hline 17 \text{ dividend} \end{array}$$

Solve and check this problem: $5\overline{)23}$ check --

If your answer is:			Go to Frame:
$\begin{array}{r} 4 \\ 5\overline{)23} \\ 20 \\ \hline 3 \end{array}$	check	$\begin{array}{r} 5 \\ \times\ 3 \\ \hline 15 \\ +\ 4 \\ \hline 19 \end{array}$	1-47 (p. 1-21)
$\begin{array}{r} 4 \\ 5\overline{)23} \\ 20 \\ \hline 3 \end{array}$		$\begin{array}{r} 5 \\ \times\ 4 \\ \hline 20 \\ +\ 3 \\ \hline 23 \end{array}$	1-55 (p. 1-27)

(1,026) (1-9)

FRAME 1-10

The commas in a whole number separate the major groups as shown in frame 1-5, page 1-10. The whole number seventy-seven million six hundred sixty-six thousand five hundred fifty-five would appear as _____.

(a. addend; addend; sum b. minuend; subtrahend; remainder) (1-23)

FRAME 1-24

Multiplication problem parts are multiplicand, multiplier, and product. The multiplicand is a number that is to be multiplied by another. The multiplier is the number doing the multiplying. The answer is called the product.

$$\begin{array}{r} 45 \\ \times\ 4 \\ \hline 180 \end{array}$$

In this problem, what is the number 4 called?

_____

FRAME 1-38

You're guessing! The problem is set up all right but there is an error in arithmetic. Take a little more time with your work. Go back to page 1-13, frame 1-34, and calculate the correct answer.

SET 7. SOLVE AND CHECK ARITHMETIC PROBLEMS WITH WHOLE NUMBERS

a.	b.	c.	d.	
447	670	125	3030	
+ 886	x 350	3/375	- 2300	(1-54)
1333	33500	3	730	
	2010	7		
	234500	6		
		15		
		15		
		0		

FRAME 1-52

All problems can be checked to see if they are correct. Checking is done by performing the opposite operation to the one that you had used. For example, when you subtract, the check is done by adding the remainder to the subtrahend. If your subtraction was correct, the result would be the MINUEND.

Look at this problem:

```
  300          check      270  remainder
- 30                     + 30  subtrahend
  270                     300  minuend (sum)
```

Solve and check this problem:

```
  455
- 50        check _____
```

If your answer is:			Go to Frame:
455	check	405	1-46 (p. 1-9)
- 50		- 50	
405		355	
455		405	1-49 (p. 1-15)
- 50		+ 50	
405		455	

(77,666,555) (1-10)

FRAME 1-11

Match the word form number in column A with the numerical form in column B by placing the letter of the word form next to the correct number.

A		B
a.	Three hundred five.	_____ 349,604
b.	Four thousand three hundred forty-five.	_____ 70,000
c.	Seventy million.	_____ 4,345
d.	Three hundred forty-nine million six hundred four.	_____ 305
		_____ 44,345
		_____ 70,000,000
		_____ 349,000,604

(multiplier) (1-24)

FRAME 1-25

The numbers that are being multiplied are also called factors. Label the problem below.

$$\begin{array}{r} 25 \\ \times\ 3 \\ \hline 75 \end{array}$$

The 25 and 3 are also called _____.

FRAME 1-39

Watch your addition. Make sure that you "carry", because that is just what you neglected in this problem. Go back to page 1-17, frame 1-36 and check the other answer. Then go to the frame with the correct answer.

(555 - 396 = 159) (1-40)

FRAME 1-53

Fine. You are now ready for multiplcation. Multiplication is actually a shortcut for addition. For example: 4 x 9 = 36 or 9 + 9 + 9 + 9 = 36. The smaller number should go on the bottom. For example, 300 x 21 = is set up and solved like this:

$$\begin{array}{r} 300 \\ \times\ 21 \\ \hline 300 \\ 600 \\ \hline 6300 \end{array}$$

Set up and solve: 2764 x 22 =

If your answer is:	Go to Frame:
$\begin{array}{r} 2764 \\ \times\ 22 \\ \hline 5528 \\ 5528 \\ \hline 11056 \end{array}$	1-48 (p. 1-13)
$\begin{array}{r} 2764 \\ \times\ 22 \\ \hline 5528 \\ 5528 \\ \hline 60808 \end{array}$	1-44 (p. 1-5)

SET 4. ROUNDING OFF WHOLE NUMBERS

(a. 305; b. 4,345; c. 70,000,000; d. 349,000,604) (1-11)

FRAME 1-12

In ROUNDING OFF whole numbers, first determine the place you want to round off. Then look at the number DIRECTLY to the right of that place. If it is 5 or more, the place you want to round off is increased by 1. For example: Round 10,869 to TENS. First look at the number to the right of the tens place. It is a 9. So the tens place (6) becomes 1 more (7) and the whole number becomes

_____.
(number)

(multiplicand; multiplier; product; factors) (1-25)

FRAME 1-26

Label the parts of each problem below.

a. 387	_____	b. 300	_____
- 20	_____	x 3	_____
367	_____	900	_____

c. The 300 and the 3 in b are called _____.

(3006 - 375 = 2631) (1-41)

FRAME 1-40

Right. Try another to make sure that you have the idea.

555 - 396 =

If your answer is:	Go to Frame:
555 - 396 259	1-43 (p. 1-3)
555 - 396 159	1-53 (p. 1-23)

(450 ÷ 5 = 90) (1-44)

FRAME 1-54

Right. You are now ready to set up and solve problems in each of the processes of addition, subtraction, multiplication, and division. Set up and solve the following problems. SHOW ALL WORK.

a. 447 + 886 =

b. 670 x 350 =

c. 375 ÷ 3 =

d. 3030 - 2300 =

Go to page 1-21, frame 1-52.

(10,870) (1-12)

FRAME 1-13

If the number to the right of the place is LESS THAN 5, the value of the place being rounded off will not change. The whole number 349 rounded to hundreds is_____.

(a. minuend; subtrahend; remainder b. multiplicand; multiplier; product c. factors) (1-26)

FRAME 1-27

Division is often thought of as determining the number of times one number is contained in another number. The answer is known as the quotient. If you were to divide 18 by 6, you might ask yourself "How many 6's are there in 18?" Your answer would be 3. The answer 3 in this problem, is called the_____.

(48 + 297 + 3 = 348) (1-34)

FRAME 1-41

Right. Now let's subtract whole numbers. Just as in addition, like units must go under like units, BUT the smaller of the numbers will go under the larger.

Set up and solve this problem. 3006 - 375 =

If your answer is:	Go to Frame:
375 - 3006 2369	1-37 (p. 1-19)
3006 - 375 2631	1-40 (p. 1-25)

$5\overline{)23}$ = 4 R 3 check: 5 x 4 = 20; 20 + 3 = 23 (1-51)

FRAME 1-55

You have now completed the solving and checking of all functions. Remember, the only one you did not do the opposite process to check was ADDITION, and then all that you do is: Invert the column of numbers and add again.

If you need more review on solving and checking, return to the page listed below that teaches what you need. Otherwise, go to page 1-7, frame 1-45.

SUBTRACTION	Frame 1-46 (p. 1-9)
MULTIPLICATION	Frame 1-49 (p. 1-15)
DIVISION	Frame 1-51 (p. 1-19)
ADDITION	Paragraph above or frame 1-49 (p. 1-15)

After you complete this review, go to page 1-7, frame 1-45.

(300) (1-13)

FRAME 1-14

ONLY the number to the right of the place you are rounding off is considered. Do not consider any other number. Round off 21,250 to thousands. _____ .

Turn back to the bottom of page 1-2.

(quotient) (1-27)

FRAME 1-28

The dividend is the number or quantity that is being divided and the divisor is the number that is divided into the dividend. In the problem $6\overline{)18}$, the 6 is the divisor and the 18 is the _____ .

Turn back to the top of page 1-3.

$$\begin{array}{r} 20047 \\ 27 \\ 598 \\ \hline 20672 \end{array}$$ (1-36)

FRAME 1-42

Now you have it. Remember that like units go under like units. You are now ready for subtraction. Look at page 1-27, frame 1-41, read the information, and work the problem.

---

a. $10\overline{)352}$ check $\begin{array}{r} 35 \\ \times\ 10 \\ \hline 350 \\ +\ \ 2 \\ \hline 352 \end{array}$

$\begin{array}{r} 35 \\ 30 \\ \hline 52 \\ 50 \\ \hline 2 \end{array}$

b. $\begin{array}{r} 12 \\ +\ 305 \\ \hline 333 \end{array}$ check $\begin{array}{r} 305 \\ +\ \ 12 \\ \hline 333 \end{array}$

c. $\begin{array}{r} 15 \\ \times\ 3 \\ \hline 45 \end{array}$ check $3\overline{)45}$ $\begin{array}{r} 15 \\ 3 \\ \hline 15 \\ 15 \\ \hline 0 \end{array}$ or $15\overline{)45}$ $\begin{array}{r} 3 \\ 45 \\ \hline 0 \end{array}$

d. $\begin{array}{r} 127 \\ -\ 39 \\ \hline 88 \end{array}$ check $\begin{array}{r} 39 \\ +\ 88 \\ \hline 127 \end{array}$ (1-45)

FRAME 1-56

This lesson may have appeared too simple for you, and perhaps it was, but it is the necessary foundation for the lessons to follow. It should also serve as a good review, since, the last time you may have studied arithmetic was some time ago. If you have learned other methods of solving mathematical problems prior to this lesson, you may use them on tests. The only thing you should remember is: BE SURE YOU ARE RIGHT. Turn to page 1-30 for a SELF TEST on the objectives of this lesson.

SELF-TEST ON INTRODUCTION TO ARITHMETIC AND WHOLE NUMBERS

1. Write, in your own words, the definition of a whole number.

_____

2. Label the place (position) of each digit in the whole number below.

(9,257,800)

9, _____

2 _____

5 _____

7, _____

8 _____

0 _____

0 _____

3. Match the words in column A with the numbers in column B by placing the letter of the numerical form next to the word form it corresponds with.

_____ Forty-nine thousand six hundred twenty-two	a. 312
_____ Ninety-nine million two	b. 490,622
_____ Three hundred twelve	c. 3,999
_____ Three thousand nine hundred ninety-nine	d. 99,000,200
	e. 49,622
	f. 99,000,002

4. Round off each of the whole numbers below.

   a. 127,009 to tens _____

   b. 2,836,999 to ten thousands _____

   c. 333,501,000 to millions _____

   d. 999 to hundreds _____

   e. 149,999 to hundred thousands _____

   f. 297 to five hundreds _____

5. Label each part of each problem below.

   a. $\begin{array}{r} 450 \\ -\ 25 \\ \hline 425 \end{array}$

   450-- _____

   25-- _____

   425-- _____

   b. $40\overline{)442}$ (11, 20, 42, 40, 2)

   40-- _____

   11-- _____

   442-- _____

   2-- _____

   c. $\begin{array}{r} 399 \\ +\ 27 \\ \hline 426 \end{array}$

   399-- _____

   27-- _____

   426-- _____

   d. $\begin{array}{r} 300 \\ \times\ 6 \\ \hline 1800 \end{array}$

   300-- _____

   6-- _____

   1800-- _____

   The 300 and the 6 are both _____.

6. Set up and solve the problems below. SHOW ALL WORK.

a. 455 x 33 =

b. 3,690 - 2,460 =

c. 44 + 275 + 9 =

d. 400 ÷ 50 =

7. Solve and check each of the problems below. SHOW ALL WORK.

a. $38\overline{)308}$ check--

b. 389
27
+ 122 check--

c. 47
x 22 check--

d. 996
- 57 check--

ANSWERS TO SELF-TEST LESSON 1

1. A whole number represents a complete unit.

2. millions
   hundred thousands
   ten thousands
   thousands
   hundreds
   tens
   units

3. e
   f
   a
   c

4. a. 127,010

   b. 2,840,000

   c. 334,000,000

   d. 1,000

   e. 100,000

   f. 500

5. a. minuend
      subtrahend
      remainder

   b. divisor
      quotient
      dividend
      remainder

   c. addend
      addend
      sum

   d. multiplicand
      multiplier
      product
      factors

6. a. $455 \times 33 = 1365 + 13650 = 15015$

b. $3{,}690 - 2{,}460 = 1{,}230$

c. $44 + 275 + 9 = 328$

d. $50\overline{)400} = 8$, $400 - 400 = 0$

7. a. $38\overline{)308} = 8$, $308 - 304 = 4$

check $38 \times 8 = 304 + 4 = 308$

b. $389 = 27 + 122 + 240$

check $122 + 27 + 389 = 538$

c. $47 \times 22 = 94 + 940 = 1034$

check $22\overline{)1034} = 47$, $1034 - 88 = 154$, $154 - 154 = 0$

or $47\overline{)1034} = 22$, $1034 - 94 = 94$, $94 - 94 = 0$

d. $996 - 57 = 939$

check $939 + 57 = 996$

LESSON 2

COMMON FRACTIONS

CREDIT HOURS ---------- 2

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Define a fraction.	2-1 (p.2-2)
2. Identify the two parts of a given fraction and explain what each part shows.	2-5 (p.2-10)
3. Identify proper fractions, improper fractions, and mixed numbers.	2-15 (p.2-30)
4. Change improper fractions to mixed numbers and mixed numbers to improper fractions.	2-23 (p.2-46)
5. Reduce fractions to their lowest terms.	2-32 (p.2-12)
6. Solve problems in addition and subtraction of fractions. Answers must be in lowest terms.	2-34 (p.2-16)
7. Solve problems in multiplication of fractions, cancelling where applicable. Answers must be in lowest terms.	2-45 (p.2-38)
8. Solve problems in division of fractions, cancelling where applicable. Answers must be in lowest terms.	2-58 (p.2-13)

SET 1. DEFINITION OF FRACTION

Frames 2-1 through 2-26 are at the top of even numbered pages.

FRAME 2-1

A fraction is a part of a whole. 3/4 is a fraction and therefore is a part of a _____.

Frames 2-27 through 2-52 are at the bottom of even numbered pages.

(4 10/11 -- if you missed this one, re-read and re-work frame 2-22 through 2-26)

FRAME 2-27

An improper fraction can be changed to a mixed number. So can a mixed number be changed to an improper fraction. Therefore, an improper fraction is interchangeable with a _____number.

(numerator; denominator (either order); divided) (2-52)

FRAME 2-53

In the problem $\frac{2}{15} \times \frac{3}{8}$, the 2 and 8 can be cancelled by dividing each by _____ and the 3 and 15 cancelled by dividing each by _____. The answer to the problem, then, is _____.

$\left[1\frac{1}{4}\right]$ (2-44)

FRAME 2-78

Right! $1\frac{1}{4}$ is the correct answer. Try another, reduce to the lowest terms. Add $\frac{1}{2} + \frac{1}{2} + \frac{4}{5} + \frac{3}{20} =$ _____

If your answer is:	Go to Frame:
$1\frac{9}{10}$	2-83 (p. 2-13)
$1\frac{19}{20}$	2-86 (p. 2-19)

(whole) (2-1)

FRAME 2-2

Part of a whole is the definition of a _____.

(mixed) (2-27)

FRAME 2-28

Changing mixed numbers to improper fractions requires three steps:

Example: Change $4\frac{3}{5}$ to an improper fraction.

Steps

(1) Multiply the whole number by the denominator of the fraction.

$4 \times 5 = 20$.

(2) Add the product to the numerator. $20 + 3 = 23$.

(3) Place the sum over the denominator of the fraction.

Then $4\frac{3}{5} =$ _____
(improper fraction)

$\left[2;\ 3;\ \frac{1}{20}\right]$ (2-53)

FRAME 2-54

In the problem $\frac{10}{13} \times \frac{26}{50} \times \frac{7}{21}$, the 10 and 50 are cancelled by dividing each by _____; the 13 and 26 are cancelled by dividing each by _____; and $\frac{7}{21}$ can be reduced to _____. Now solve the problem, showing your cancellation. $\frac{10}{13} \times \frac{26}{50} \times \frac{7}{21}$ = _____.

$\left[\frac{3}{7}\right]$ (2-32)

FRAME 2-79

$\frac{3}{7}$ is correct. One (1) is the only number that divides evenly into both 3 and 7.

Let's try a larger fraction. Reduce $\frac{42}{54}$ to its lowest terms.

If your answer is:	Go to Frame:
$\frac{21}{27}$	2-62 (p. 2-21)
$\frac{7}{9}$	2-65 (p. 2-27)

(fraction) (2-2)

FRAME 2-3

The definition of a fraction is stated as: _____ of a _____.

$\left[\frac{23}{5}\right]$ (2-28)

FRAME 2-29

Change $12\frac{2}{3}$ to an improper fraction.

If your answer is	Go to FRAME
$\frac{36}{3}$	2-61 (p. 2-19)
$\frac{38}{2}$	2-64 (p. 2-25)
$\frac{38}{3}$	2-67 (p. 2-31)

$$\left[10;\ 13;\ \frac{1}{3};\ \frac{\cancel{10}^{1}}{\cancel{15}_{1}} \times \frac{\cancel{26}^{2}}{\cancel{50}_{5}} \times \frac{\cancel{7}^{1}}{\cancel{21}_{3}} = \frac{2}{15}\right] \quad (2\text{-}54)$$

FRAME 2-55

Solve the following problems, using cancellation where applicable. Reduce answers to lowest terms.

a. $\frac{2}{5} \times \frac{3}{10} \times \frac{7}{9} =$

b. $\frac{12}{16} \times \frac{8}{24} \times \frac{8}{10} =$

FRAME 2-80

No. Not quite. Your addition is correct but you must have overlooked the "reduce answers to lowest terms." Go back to page 2-36, frame 2-44, reduce, and pick the correct answer.

(part; whole) (2-3)

FRAME 2-4

Define a fraction.

_____ _____ _____ _____

FRAME 2-30

If you came to this page directly from the previous page, you have not followed the directions given in the previous frame. From this point (unless otherwise directed) in the lesson, you will proceed by the branching method. Do Not read the frames in sequence, but after selecting an answer, refer to the proper page or frame as directed. Return to frame 2-29, check your answer, and refer to the page as directed.

[a. $\frac{7}{75}$; b. $\frac{1}{5}$] (2-55)

FRAME 2-56

In order to multiply fractions and mixed numbers, the mixed numbers must be changed to improper fractions.

Examples: $2\frac{1}{2} \times \frac{3}{8} \times 1\frac{1}{3}$ will be changed to

$$\frac{5}{2} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{5}{4} \text{ reduced is } 1\frac{1}{4}$$

Solve the following problems, using cancellation where applicable, and reduce answers to lowest terms:

a. $3\frac{1}{3} \times 5\frac{1}{2} \times \frac{9}{10} =$

b. $4\frac{1}{2} \times 3\frac{1}{3} \times 2\frac{2}{8} =$

c. $\frac{3}{4}$ of 80 =

FRAME 2-81

$\frac{6}{15}$ is wrong. You borrowed one (1) from 16, which gave you the fraction $\frac{15}{15}$, but now you must add $\frac{15}{15} + \frac{8}{15}$, then do your subtraction. Return to page 2-17, frame 2-85, rework the problem, and select another answer.

SET 2. PARTS OF A FRACTION

(part of a whole) (2-4)

FRAME 2-5

Fractions have two parts -- a numerator (above the line) and a denominator (below the line).

Example: $\frac{3}{8}$ -- numerator
8 -- denominator

In the fraction $\frac{2}{3}$, the number 3 below the line is the _____ and the number 2 above the line is the _____.

$\left[7\frac{1}{4} = \frac{29}{4};\ \frac{29}{4} = 7\frac{1}{4}\right]$ (2-69)

FRAME 2-31

Change each of the following improper fractions to mixed numbers and the mixed numbers to improper fractions:

a. $1\frac{4}{9}$

b. $\frac{21}{8}$

c. $10\frac{11}{12}$

d. $\frac{49}{3}$

[a. $16\frac{1}{2}$; b. $33\frac{3}{4}$; c. 60] (2-56)

FRAME 2-57

Solve the problems below, cancelling where applicable, and reduce answers to lowest terms.

a. $\frac{3}{5}$ of $2\frac{5}{8}$ =

b. $3\frac{1}{2} \times 2\frac{1}{4} \times \frac{2}{3}$ =

c. $\frac{1}{6}$ of 24 =

d. $2\frac{1}{8} \times 3\frac{3}{4} \times 1\frac{1}{3}$ =

FRAME 2-82

You have the correct fraction but made a mistake in the addition of whole numbers. Now return to page 2-25, frame 2-89, and work the problem again. Do not just pick the other answer without first re working the problem to find your error.

(denominator; numerator) (2-5)

FRAME 2-6

All fractions have denominators and numerators. In the fractions $\frac{2}{3}$ and $\frac{11}{12}$, the 3 and 12 are _____ and the 2 and 11 are _____.

SET 5. REDUCE FRACTION TO LOWEST TERMS

[a. $\frac{13}{9}$; b. $2\frac{5}{8}$; c. $\frac{131}{12}$; d. $16\frac{1}{3}$] (2-31)

FRAME 2-32

A fraction is in its <u>lowest terms</u> when the number one (1) is the <u>only</u> number that divides evenly into both the numerator and denominator. (NOTE: Dividing both the numerator and denominator by the same number does not change the value of the fraction.) Select the fraction below that is in its lowest terms.

If your answer is:	Go to Frame:
$\frac{2}{4}$	2-74 (p. 2-45)
$\frac{6}{9}$	2-77 (p. 2-51)
$\frac{3}{7}$	2-79 (p. 2-5)

SET 7. DIVISION OF FRACTIONS

[a. $1\frac{23}{40}$; b. $5\frac{1}{4}$; c. 4; d. $10\frac{5}{8}$] (2-57)

FRAME 2-58

Dividing common fractions requires two steps:

Example: $\frac{2}{7} \div \frac{1}{3} =$

Dividend Divisor

(1) Obtain reciprocal of divisor -- $\frac{3}{1}$.
(invert divisor)

(2) Multiply the dividend by the reciprocal of the divisor -- $\frac{2}{7} \times \frac{3}{1} = \frac{6}{7}$.

Then $\frac{2}{7} \div \frac{1}{3} =$ _____.

FRAME 2-83

Incorrect. You've made a mistake someplace in changing fractions to equivalent fractions of the same denominator. Return to page 2-30, frame 2-41, re-read the rule, then go back to page 2-3, frame 2-78, and choose the other answer.

(denominators; numerators) (2-6)

FRAME 2-7

The denominator tells how many equal parts the whole has been divided into. In the fraction $\frac{9}{10}$, the denominator indicates the whole has been divided into _____ equal parts.

(You came from frame 2-75)

FRAME 2-33

Reduce each of the following fractions to lowest terms:

a. $\frac{12}{4} =$

b. $\frac{21}{49} =$

c. $\frac{64}{72} =$

d. $\frac{17}{51} =$

$\left[\frac{6}{7}\right]$ (2-58)

FRAME 2-59

Fill in the steps to find $\frac{5}{9} \div \frac{3}{4}$.

(1) Obtain reciprocal of divisor (invert the divisor) _____

(2) Multiply the dividend by the reciprocal of the divisor _____

Then $\frac{5}{9} \div \frac{3}{4}$ = _____.

FRAME 2-84

$\frac{1}{4}$ is wrong. You did not obtain the reciprocal of the divisor.

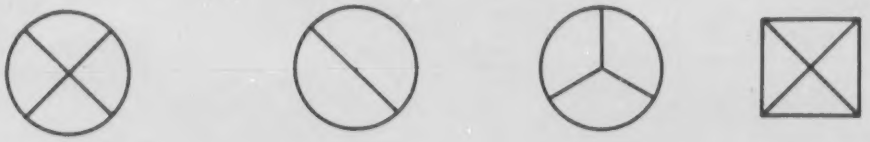
$\frac{2}{3}$ inverted is $\frac{3}{2}$ and the reciprocal of $\frac{2}{3}$ is also $\frac{3}{2}$.

Go back to page 2-23, frame 2-63, re-work the problem, and select the correct answer.

(10) (2-7)

FRAME 2-8

The shapes in the figure below have been divided into equal parts. Write the number that would be used as the denominator of a fraction. (The number of parts into which the whole shape has been divided.)



a. _____ b. _____ c. _____ d. _____

SET 6. ADDITION AND SUBTRACTION OF FRACTIONS

[a. 3 or $\frac{3}{1}$; b. $\frac{3}{7}$; c. $\frac{8}{9}$; d. $\frac{1}{3}$] (2-33)

FRAME 2-34

To <u>add</u> or <u>subtract</u> fractions, they must be <u>like</u> fractions. <u>Like</u> fractions have the same number for a denominator.

$\frac{7}{12} + \frac{5}{12}$ or $\frac{7}{12} - \frac{5}{12}$ are like fractions because they have the same number for a _____.

$\left[\frac{4}{3};\ \frac{5}{9} \times \frac{4}{3};\ \frac{20}{27}\right]$ (2-59)

FRAME 2-60

Solve this problem: $\frac{3}{10} \div \frac{3}{4}$

If your answer is:	Go to Frame:
$\frac{2}{5}$	2-63 (p. 2-23)
$\frac{9}{40}$	2-66 (p. 2-29)
$\frac{12}{30}$	2-101 (p. 2-49)

$\left[13\ \frac{5}{9}\right]$ (2-89)

FRAME 2-85

Very good. Work the following problem by subtracting mixed numbers. Reduce answer to lowest term. $16\ \frac{8}{15} - 15\ \frac{3}{5} =$ _____

If your answer is:	Go to Frame:
$1\ \frac{14}{15}$	2-88 (p. 2-23)
$\frac{14}{15}$	2-90 (p. 2-27)
$\frac{6}{15}$	2-81 (p. 2-9)
can't be solved	2-94 (p. 2-35)

(a. 4; b. 2; c. 3; d. 4) (2-8)

FRAME 2-9

In the fraction below, circle the denominator and explain what it indicates:

$\frac{15}{16}$ _____ _____ _____ _____ _____

_____ _____ _____ _____ _____

_____

(denominator) (2-34)

FRAME 2-35

Fractions must have like (common) denominators before you can _____ or _____ them.

FRAME 2-61

Wrong! 12 x 3 = 36, but you must now do step 2. Add this product (36) to the numerator; retain the denominator to get the improper fraction. Go back to page 2-6, frame 2-29, and select another answer.

$\left[1\ \frac{19}{20}\right]$ (2-78)

FRAME 2-86

Good. $1\ \frac{19}{20}$ is correct. Now try one on subtraction and reduce answer to lowest terms. $\frac{4}{13} - \frac{3}{39} =$ _____.

If your answer is:	Go to Frame:
$\frac{3}{13}$	2-89 (p. 2-25)
$\frac{3}{39}$	2-99 (p. 2-45)

(16) denominator. Tells (indicates) how many parts the whole has been divided into) (2-9)

FRAME 2-10

The numerator (number above the line) of a fraction shows "how many parts of the whole are being considered." In the fraction $\frac{2}{3}$, the numerator indicates that _____ parts of the whole are being considered and the denominator indicates that the whole has been divided into _____ equal parts.

(add; subtract (any order)) (2-37)

FRAME 2-36

When fractions have common denominators, you add or subtract numerators and retain the common denominator.

Example: $\frac{7}{12} + \frac{5}{12} = \frac{12}{12}$ reduced = 1

Then $\frac{7}{12} - \frac{5}{12}$ = _____ reduced = _____.

FRAME 2-62

Nope! You will still have to go to lower terms. You reduced by dividing two into the numerator and denominator but you must now find a number to further reduce $\frac{21}{27}$ and then you'll have it. Return to page 2-5, frame 2-79, select the other answer, and continue.

FRAME 2-87

$6\frac{29}{36}$ is incorrect. Again you forgot to invert the divisor. The divisor $1\frac{1}{6}$ is changed to $\frac{7}{6}$ and inverted is $\frac{6}{7}$. Now go back to page 2-39, frame 2-96, and select another answer.

(2; 3) (2-10)

FRAME 2-11

In the fraction $\frac{13}{14}$, the number of parts being considered is _____ and the part of the fraction that tells us this is called the _____.

$\left[\frac{2}{12}; \frac{1}{6}\right]$ (2-36)

FRAME 2-37

Before fractions with unlike denominators can be added or subtracted, they must be changed to their lowest common denominator (LCD). LCD is the lowest number that is divisible by each denominator.

Example: $\frac{2}{5} + \frac{1}{20}$ or $\frac{2}{5} - \frac{1}{20}$

The lowest number divisible by each denominator is 20; therefore, 20 is the _____ _____ _____ _____.

$\left[\frac{2}{5}\right]$ (2-60)

FRAME 2-63

$\frac{2}{5}$ is the correct answer.

Now try another problem. $\frac{3}{8} \div \frac{2}{3} =$ _____

If your answer is:	Go to Frame:
$\frac{1}{4}$	2-84 (p. 2-15)
4 or $\frac{4}{1}$	2-93 (p. 2-33)
$\frac{9}{16}$	2-96 (p. 2-39)

FRAME 2-88

You've forgotten the rule on borrowing. $16\frac{8}{15} = 16\frac{8}{15} = 15\frac{23}{15}$

$-15\frac{3}{5} = -15\frac{9}{15} = -15\frac{9}{15}$

You cannot subtract $\frac{9}{15}$ from $\frac{8}{15}$, so you have to borrow a whole number (1). $1 = \frac{15}{15}$, which you now add to the $\frac{8}{15}$. Don't forget now that you borrowed a whole number from 16. Go back to page 2-17, frame 2-85. Rework the problem and select the correct answer.

(13, numerator) (2-11)

FRAME 2-12

The number of parts being considered is indicated by the _____ of a fraction.

(least or lowest common denominator or LCD) (2-37)

FRAME 2-38

Again, the lowest number divisible by each denominator of fractions to be added or subtracted is called the _____ _____ _____.

FRAME 2-64

Wrong! Multiplication and addition are correct but you must place this sum over the <u>denominator</u> of the fraction. Return to page 2-6, frame 2-29, and select another answer.

$\left[\frac{3}{13}\right]$ (2-86)

FRAME 2-89

Good. Now for the rule for adding and subtracting mixed numbers: 1. Change fractions to <u>like</u> fractions (LCD). 2. Add or subtract the fractions. 3. Add or subtract the whole numbers. 4. Reduce answers to lowest terms. Example: $1\frac{1}{3} + 3\frac{11}{12}$ and $7\frac{1}{2} - 4\frac{1}{5}$.

$$1\frac{1}{3} = 1\frac{4}{12}$$
$$+ \quad 3\frac{11}{12} = 3\frac{11}{12}$$
$$4\frac{15}{12} = 4 + 1\frac{3}{12} = 5\frac{1}{4}$$

$$7\frac{1}{2} = 7\frac{5}{10}$$
$$- 4\frac{1}{5} = - 4\frac{2}{10}$$
$$3\frac{3}{10}$$

Now add these fractions: $7\frac{1}{9} + 6\frac{5}{18} + \frac{1}{6} =$ _____.

If your answer is:	Go to Frame:
$14\frac{5}{9}$	2-82 (p. 2-11)
$13\frac{5}{9}$	2-85 (p. 2-17)

(numerator) (2-12)

FRAME 2-13

Under the shapes in the figure below, write the fractions. The number of parts being considered are shaded.

a. _____ b. _____ c. _____ d. _____

(LCD) (2-38)

FRAME 2-39

Determine the lowest common denominator (LCD) for these fractions:

$\frac{1}{2} + \frac{1}{4}$, the LCD is _____.

$\frac{2}{7} - \frac{1}{42}$, the LCD is _____.

$\left[\frac{7}{9}\right]$ (2-79)

FRAME 2-65

Right! Now try this. Reduce $\frac{14}{56}$ to its lowest term.

If your answer is:	Go to Frame:
$\frac{1}{4}$	2-68 (p. 2-33)
$\frac{7}{28}$	2-70 (p. 2-37)

$\left[\frac{14}{15}\right]$ (2-85)

FRAME 2-90

Very good. The idea here was to see if you remember how to borrow. Solve the addition and subtraction problems below. Answers must be in lowest terms.

a. $\frac{1}{21} + \frac{4}{7} + \frac{2}{3} =$

b. $3\frac{3}{8} - 2\frac{1}{4} =$

c. $11\frac{1}{8} + 1\frac{3}{16} + \frac{1}{2} + \frac{3}{4} =$

d. $14\frac{1}{6} - 12\frac{5}{12} =$

Go to page 2-38, frame 2-45 to check answers and continue from there.

[a. $\frac{1}{3}$; b. $\frac{3}{4}$; c. $\frac{2}{3}$; d. $\frac{1}{4}$] (2-13)

FRAME 2-14

In the fraction below, write what each number is called and what it indicates: $\frac{6}{7}$

6 -- _____. _____ _____ _____ _____ _____

_____ _____ _____ _____ _____.

7 -- _____. _____ _____ _____ _____

_____ _____ _____ _____ _____ _____

_____.

(4; 42) (2-89)

FRAME 2-40

Find the LCD for the fractions below:

a. $\frac{5}{8} + \frac{1}{16} + \frac{1}{4}$, the LCD is _____.

b. $\frac{4}{7} - \frac{1}{49}$, the LCD is _____.

FRAME 2-66

No! You forgot to obtain the reciprocal of the divisor (invert the divisor), before you multiplied. Go back to frame 2-58, review the procedure again, then rework the problem from page 2-17, frame 2-60 again and select the correct answer.

---

(5) (2-96)

FRAME 2-91

5 is the correct answer. Try one more.

$$5\frac{4}{7} \div 3 =$$

If your answer is:	Go to Frame:
$1\frac{6}{7}$	2-102 (p. 2-51)
$16\frac{5}{7}$	2-97 (p. 2-41)
$\frac{13}{7}$	2-95 (p. 2-37)

SET 3. TYPES OF COMMON FRACTIONS

(6-- numerator. Indicates how many parts of the whole are being considered. 7-- denominator. Indicates how many equal parts the whole has been divided into) (2-14)

FRAME 2-15

There are three types of common fractions -- proper, improper, and mixed numbers. The three types of common fractions are mixed numbers, _____ and _____ fractions.

(a. 16; b. 49) (2-40)

FRAME 2-41

After the LCD has been determined, change all fractions to equivalent fractions of the same denominator; then add or subtract. Example: $\frac{2}{7} + \frac{1}{42}$, the LCD is 42. To change $\frac{2}{7}$ to LCD 42: Divide 7 into 42; the quotient is 6. Multiply 6 by the numerator 2 and place the product (12) over the LCD. $\frac{2}{7} = \frac{12}{42}$. Now we can add. $\frac{12}{42} + \frac{1}{42} = \frac{13}{42}$ reduced is $\frac{13}{42}$

Change the fractions below to their LCD.

a. $\frac{1}{3} + \frac{5}{6} + \frac{1}{12} =$ _____ + _____ + _____

b. $\frac{4}{5} - \frac{1}{3} =$ _____ - _____.

$\left[\frac{38}{3}\right]$ (2-29)

FRAME 2-67

Correct. Now change $15\frac{1}{5}$ to an improper fraction.

If your answer is:	Go to Frame:
$\frac{76}{5}$	2-69 (p. 2-35)
$\frac{75}{5}$	2-71 (p. 2-39)

FRAME 2-92

Not quite. $\frac{35}{7}$ is an improper fraction and for the answer to be completely correct (lowest terms), you must now change your answer to a mixed number. Return to page 2-39, frame 2-96, recheck your work, and reduce answer to lowest terms.

(proper; improper (either order)) (2-15)

FRAME 2-16

The difference between proper and improper fractions is the size of the numerator. The numerator of an improper fraction is always the same as or larger than the denominator; therefore, in a proper fraction, the numerator is _____ than the denominator.

[a. $\frac{4}{12} + \frac{10}{12} + \frac{1}{12}$; b. $\frac{12}{15} - \frac{5}{15}$] (2-41)

FRAME 2-42

Find the LCD and change the fractions below to equivalent fractions: a. $\frac{1}{9} + \frac{1}{81} + \frac{2}{3} =$ _____ + _____ + _____

b. $\frac{4}{5} - \frac{5}{8} =$ _____ - _____

$\left[\frac{1}{4}\right]$ (2-65)

FRAME 2-68

Good! You might have started with dividing by two (2) and doing several steps, but 14 divides into 14 and 56 evenly. To reduce an improper fraction such as $\frac{8}{4}$ or $\frac{9}{5}$, you simply divide the denominator into the numerator. Reduce $\frac{9}{5}$ to its lowest terms.

If your answer is:	Go to Frame:
$\frac{9}{5}$	2-72 (p. 2-41)
$1\frac{4}{5}$	2-75 (p. 2-47)

FRAME 2-93

4 or $\frac{4}{1}$ is incorrect. You obtained the reciprocal of the dividend. You're to obtain the reciprocal of the divisor and then proceed as in multiplication. Now go to page 2-23, frame 2-63, rework the problem, and select the correct answer.

(smaller (less than)) (2-16)

FRAME 2-17

$\frac{7}{8}$ is a proper fraction because the _____ is _____ than the denominator.

[a. $\frac{9}{81} + \frac{1}{81} + \frac{54}{81}$; b. $\frac{32}{40} - \frac{25}{40}$] (2-42)

FRAME 2-43

The rule again for adding and subtracting fractions: (1) Change fractions to common denominators. (2) Add or subtract numerators. (3) Keep common denominator. (4) Reduce answers to lowest terms. Below are the LCD problems from the last frame. Complete the problems.

a. $\frac{9}{81} + \frac{1}{81} + \frac{54}{81}$ = _____ reduced is _____

b. $\frac{32}{40} - \frac{25}{40}$ = _____ reduced is _____

$\left[\frac{76}{5}\right]$ (2-67)

FRAME 2-69

Right! $\frac{76}{5}$ is correct. You can check your answers by changing the improper fraction back to the mixed number. Change $7\frac{1}{4}$ to an improper fraction and check your answer.

$7\frac{1}{4} =$ _____ (improper fraction) $=$ _____ (mixed number)

Go to page 2-10, frame 2-31 to check answer and continue from there.

FRAME 2-94

You've forgotten the rule on borrowing. True, you can't subtract $15\frac{9}{15}$ from $16\frac{8}{15}$ unless you borrow. Why not take one (1) from 16 and add the fraction $\frac{15}{15}$ to $\frac{8}{15}$? Now you can subtract, but don't forget the one (1) you borrowed. Go back to page 2-17, frame 2-85, rework the problem, and select another answer.

(numerator; smaller (less)) (2-17)

FRAME 2-18

$\frac{8}{7}$ and $\frac{8}{8}$ are improper fractions because the _____ are _____ _____ _____ than the denominators.

[$\frac{64}{81}$ reduced is $\frac{64}{81}$; $\frac{7}{40}$ reduced is $\frac{7}{40}$] (2-43)

FRAME 2-44

Do you understand all the steps now? Solve this problem and reduce answer to lowest terms: $\frac{1}{28} + \frac{6}{7} + \frac{5}{14} =$ _____

If your answer is:	Go to Frame:
$1\frac{1}{4}$	2-78 (p. 2-3)
$1\frac{7}{28}$	2-80 (p. 2-7)
$\frac{12}{28}$	2-76 (p. 2-49)
$\frac{3}{7}$	2-73 (p. 2-43)

FRAME 2-70

You reduced -- but not to the lowest terms. Return to page 2-27, frame 2-65, and find the number that will reduce the $\frac{7}{28}$ and then you'll have the correct answer that will allow you to continue.

FRAME 2-95

$\frac{13}{7}$ is unacceptable, because the answer has not been reduced to its lowest terms. Return to page 2-29, frame 2-91, and select the correct answer that is in its lowest terms.

(numerators; are same as or greater (are the same as or larger))

(2-18)

FRAME 2-19

In the list below, place a "P" by the proper fractions and "I" by the improper fractions.

a. $\frac{12}{17}$

b. $\frac{9}{7}$

c. $\frac{4}{5}$

d. $\frac{12}{12}$

SET 7. MULTIPLICATION OF FRACTIONS

[You came from Frame 90: a. $1\frac{2}{7}$; b. $1\frac{1}{8}$; c. $13\frac{9}{16}$; d. $1\frac{3}{4}$]

FRAME 2-45

When multiplying two or more fractions, multiply numerators of the fractions to obtain numerator of the product. To obtain the numerator of the product in the problem $\frac{2}{3} \times \frac{2}{3}$, multiply _____ (number)

times _____ (number) .

FRAME 2-71

Wrong! You forgot to add the numerator to the product of the whole number times the denominator. If you now see your error, go back to page 2-31, frame 2-67, and select the other answer and follow directions. If you need the rule again, return to page 2-4, frame 2-28, and start again from there.

$\left[\frac{9}{16}\right]$ (2-63)

FRAME 2-96

$\frac{9}{16}$ is correct. Dividing with mixed numbers requires three steps: (1) Change the mixed number or mixed numbers to improper fractions. (2) Obtain the reciprocal of the divisor (invert divisor). (3) Multiply the dividend by the reciprocal of the divisor. Try this problem: $5\frac{5}{6} \div 1\frac{1}{6} =$

If your answer is:	Go to Frame:
5	2-91 (p. 2-29)
$6\frac{29}{36}$	2-87 (p. 2-21)
$\frac{35}{7}$	2-92 (p. 2-31)

(a. P; b. I; c. P; d. I) (2-19)

FRAME 2-20

A mixed number is a whole number combined with a proper fraction. $3\frac{5}{6}$ is a whole number (3) and a proper fraction $\frac{(5)}{(6)}$; therefore, $3\frac{5}{6}$ is a _____ _____.

(2; 2) (2-45)

FRAME 2-46

Like the numerator, the denominator of the product is obtained by multiplying the denominators of the fractions. In the problem $\frac{2}{3} \times \frac{4}{5}$, the numerator of the product is obtained by multiplying _____ (number) times _____ (number) and the denominator is obtained by multiplying _____ (number) times _____ (number).

FRAME 2-72

No... To reduce an improper fraction, you simply change it to a whole number or to a whole number and a fraction (mixed number) by dividing the numerator by the denominator. Now go back to page 2-33, frame 2-68, and reduce properly.

FRAME 2-97

No! Does it sound reasonable that 3 is contained in $5\frac{4}{7}$ ---- 16 and $\frac{5}{7}$ times? You forgot to obtain the reciprocal of the divisor before you multiplied. Go back to page 2-29, frame 2-91, invert the divisor, multiply, and then select the correct answer.

(mixed number) (2-20)

FRAME 2-21

To review definitions, match the following types of fractions with the correct statement or statements by writing the letter of the statement by the number of the fraction. All letters are to be used.

1. Proper fraction
2. Mixed number
3. Improper fraction

A. Numerator greater than the denominator

B. Numerator less than the denominator

C. Whole number and a proper fraction

D. Numerator equal to denominator

(2 x 4; 3 x 5) (2-46)

FRAME 2-47

The rule, then, for multiplying fractions is: "Multiply numerators of the fractions to obtain the _____ of the product and multiply the denominators to obtain the _____ of the product." Solve this problem:

$$\frac{2}{3} \times \frac{2}{5} = _____.$$

FRAME 2-73

Negative. You have simply added numerators, retained highest denominator, and reduced. You must change to equivalent fractions. Re-read rule on page 2-30, frame 2-41, and rework problem from page 2-34, frame 2-43 again.

[a. $\frac{5}{6}$; b. $3\frac{1}{5}$; c. $\frac{13}{27}$; d. $\frac{1}{9}$] (2-102)

FRAME 2-98

If you had any answers other than those above, you must rework the problem(s) in frame 2-102, page 2-51. When you've gotten all correct, solve these problems:

a. $5\frac{2}{3} \div 9\frac{5}{9} =$

b. $5\frac{2}{5} \times 2\frac{1}{4} \times 4\frac{2}{3} =$

c. $21\frac{1}{16} + 9\frac{3}{8} + 8\frac{1}{2} + \frac{3}{4} =$

d. $3\frac{3}{16} - 1\frac{3}{4} =$

Go to page 2-47, frame 2-100 for answers.

(B, 1; C, 2; D and A, 3) (2-21)

FRAME 2-22

In the list below, place a "P" by the proper fractions, an "I" by the improper fractions, and a "M" by the mixed numbers.

a. $3\frac{1}{2}$

b. $\frac{9}{5}$

c. $12\frac{2}{3}$

d. $\frac{3}{4}$

e. $\frac{22}{29}$

f. $\frac{7}{7}$

g. $\frac{79}{75}$

[numerator; denominator; $\frac{4}{15}$] (2-47)

FRAME 2-48

The word "of" is sometimes used in place of the multiplication sign "x". $\frac{2}{3}$ of 15 = 10 can be written as $\frac{2}{3} \times \frac{15}{1} = \frac{30}{3} = 10$. Solve this problem and reduce: $\frac{5}{8}$ of 40 = _____ reduced is _____.

FRAME 2-74

No! $\frac{2}{4}$ can be reduced to $\frac{1}{2}$ by dividing two (2) into both the numerator and denominator. Remember the rule, a fraction is in its lowest terms only when the number one (1) is the only number that divides evenly into both the numerator and denominator. Return to page 2-12, frame 2-32 and select the correct answer.

FRAME 2-99

Never! The only way you could have arrived at this answer was to have reduced the numerator and not the denominator. Return to page 2-19, frame 2-86, work the problem again, and select the correct answer.

SET 4. CONVERSION OF IMPROPER FRACTION AND MIXED NUMBERS

(a. M; b. I; c. M; d. P; e. P; f. I; g. I) (2-22)

FRAME 2-23

In improper fraction can be changed to a mixed number by dividing the denominator into the numerator. The fraction $\frac{21}{10}$ can be changed to a mixed number by dividing the numerator _____ (number) by _____ (number), the denominator.

[$\frac{200}{8}$; reduced is 25] (2-48)

FRAME 2-49

If the problem contains more than two fractions, multiply all the numerators and multiply all the denominators.

Example: $\frac{2}{5} \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{4} = \frac{4}{180}$ reduced $\frac{1}{45}$

Solve this problem:

$\frac{3}{5} \times \frac{4}{7} \times \frac{1}{2}$ = _____ reduced = _____.

$\left[1\frac{4}{5}\right]$ (2-68)

FRAME 2-75

$1\frac{4}{5}$ is correct. If we ask you to reduce the fraction $\frac{8}{4}$, would you answer 2? You would have been correct there, too. Now turn to page 2-14, frame 2-33, and continue the program.

$\left[\text{a. } \frac{51}{86} \text{ ; b. } 56\frac{7}{10} \text{ ; c. } 39\frac{11}{16} \text{ ; d. } 1\frac{7}{16}\right]$ (2-97 p. 2-43)

FRAME 2-100

If you missed any problem, you must rework and recheck. After all problems are correct, read the rules again that are on the pages listed below.

Problem:	Go to Frame:
a. (division)	2-96 (p. 2-39)
b. (multiplication)	2-56 (p. 2-9)
c. (addition)	2-89 (p. 2-25)
d. (subtraction and borrowing)	2-88 (p. 2-23)

After you've read the rules, go to the Self-Test on page 2-53.

(21; 10) (2-23)

FRAME 2-24

To change the improper fraction $\frac{21}{10}$ to a mixed number, follow two steps: Divide the numerator by the denominator to get the whole number

$$10\overline{)21}\quad \begin{array}{l} 2 \\ \\ 20 \\ 1 \end{array} \begin{array}{l} \text{the whole number} \\ \\ \\ \text{the remainder} \end{array}$$

(2) Place the remainder over the denominator to get the proper fraction: $\frac{1}{10}$ the proper fraction. Then $\frac{21}{10} =$ _____ (mixed number)

$\left[\frac{12}{70}\text{ ; reduced } = \frac{6}{35}\right]$ (2-49)

FRAME 2-50

"Cancellation" is a "short cut" used in multiplying fractions. The short cut in multiplying fractions is called _____.

FRAME 2-76

No. You've added numerators but have not changed fractions to equivalent fractions. Read rule again on page 2-30, frame 2-41, then rework problem on page 2-36, frame 2-44 and select another answer.

FRAME 2-101

Not quite right. You must not have cancelled the 3's after obtaining the reciprocal of the divisor and you haven't reduced to the lowest terms. Go back to page 2-17, frame 2-60, and correct your mistake. Then select the correct answer.

$\left[2\frac{1}{10}\right]$ (2-24)

FRAME 2-25

Now change the improper fraction $\frac{26}{5}$ to a mixed number. Show your work.

(mixed number)

(cancellation) (2-50)

FRAME 2-51

Cancellation is much the same as reducing. The first step is to select a numerator and denominator that can be divided evenly by the same number and then divide them by that number. The problem $\frac{5}{10} \times \frac{2}{5} \times \frac{4}{10}$ can be reduced to: $\frac{\cancel{5}^{1}}{\cancel{10}_{5}} \times \frac{\cancel{2}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{10}_{5}}$. The next step is to multiply the numerators and the denominators $\frac{1}{5} \times \frac{1}{1} \times \frac{2}{5} = \frac{2}{25}$ reduced is $\frac{2}{25}$. Solve the problem below by cancellation. Show work.

$\frac{5}{8} \times \frac{4}{7} \times \frac{1}{5} =$ _____.

FRAME 2-77

Wrong! $\frac{6}{9}$ can be further reduced. Three (3) is the largest number that divides evenly into both the numerator (6) and the denominator (9). $\frac{6}{9}$, then, reduced to lowest possible terms, is $\frac{2}{3}$. Now return to page 2-12, frame 2-32, and select the correct answer.

$\left[1\frac{6}{7}\right]$ (2-91)

FRAME 2-102

$1\frac{6}{7}$ is correct. Divide the following fractions and reduce answers to lowest terms:

a. $\frac{5}{8} \div \frac{3}{4} =$

b. $22 \div 6\frac{7}{8} =$

c. $2\frac{1}{6} \div 4\frac{1}{2} =$

d. $\frac{8}{21} \div 3\frac{3}{7} =$

Turn to page 2-43, frame 2-98 to check your answers.

$$\left[\begin{array}{r} 5 \\ 5\overline{)\,26} \\ \underline{25} \\ 1 \end{array} ;\ 5\frac{1}{5}\right] \quad (2\text{-}25)$$

FRAME 2-26

Try another. Change $\frac{54}{11}$ to a mixed number.

_____ = _____.

Turn back to the bottom of page 2-2.

$$\left[\frac{\cancel{2}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{7} \times \frac{1}{\cancel{5}_{1}} = \frac{1}{14}\right] \quad (2\text{-}51)$$

FRAME 2-52

When you use the cancellation method, the basic principle is: Dividing both the numerator and the denominator by the same number does not change the value of a fraction. The value of a fraction is not changed when the _____ and the _____ are _____ by the same number.

Turn back to the top of page 2-3.

SELF-TEST ON FRACTIONS

1. Write the definition of a fraction.

_____

2. Identify the two parts of the fraction $\frac{7}{8}$ and explain what each part shows.

7-- _____

_____

8-- _____

_____

3. Identify the proper fractions, the improper fractions, and the mixed numbers in the following list by placing a "P" by the proper fractions, an "I" by the improper fractions, and a "M" by the mixed number.

a. $\frac{15}{16}$

b. $\frac{19}{17}$

c. $2\frac{4}{5}$

d. $\frac{9}{7}$

e. $77\frac{2}{3}$

f. $\frac{300}{299}$

g. $\frac{10}{11}$

h. $\frac{7}{12}$

i. $6\frac{3}{7}$

j. $\frac{5}{6}$

4. Change the mixed numbers to improper fractions and the improper fractions to mixed numbers.

a. $3\frac{2}{3}$

b. $\frac{11}{10}$

c. $12\frac{4}{5}$

d. $\frac{19}{15}$

e. $7\frac{7}{8}$

5. Reduce the following fractions to their lowest terms:

a. $\frac{18}{81}$

b. $\frac{9}{12}$

c. $\frac{21}{63}$

d. $\frac{3}{7}$

e. $\frac{14}{21}$

f. $\frac{16}{64}$

6. Solve the following addition and subtraction problems. Reduce answers to lowest terms.

a. $\frac{1}{2} + \frac{1}{2} =$

b. $\frac{5}{7} - \frac{2}{3} =$

c. $\frac{3}{8} + \frac{3}{4} =$

d. $2\frac{3}{8} - 1\frac{5}{8} =$

e. $6\frac{7}{10} - 4\frac{4}{5} =$

f. $11\frac{3}{4} + 19\frac{5}{8} + 9\frac{1}{2} + \frac{3}{16} =$

7. Multiply the following fractions, cancelling where applicable. Reduce answers to lowest terms.

a. $\frac{1}{2} \times \frac{3}{4} \times \frac{2}{3} =$

b. $4\frac{2}{3} \times 5\frac{1}{4} \times 2\frac{2}{3} =$

c. $\frac{3}{4} \times 5\frac{1}{2} =$

d. $\frac{1}{8}$ of 16 =

8. Divide the following fractions, cancelling where applicable. Reduce answers to lowest terms.

a. $\frac{7}{8} \div \frac{7}{16} =$

b. $15 \div 4\frac{1}{5} =$

c. $4\frac{2}{3} \div 12\frac{4}{9} =$

d. $\frac{4}{5} \div 2\frac{7}{15} =$

ANSWERS TO SELF-TEST LESSON 2

1. A fraction is part of a whole.

2. 7-- Numerator. Indicates how many parts of the whole are being considered.

   8-- Denominator. Indicates how many equal parts the whole has been divided into.

3. a. P
   b. I
   c. M
   d. I
   e. M
   f. I
   g. P
   h. P
   i. M
   j. P

4. a. $3\frac{2}{3}$ $3 \times 3 = 9$ $9 + 2 = 11$ $\frac{11}{3}$

   b. $\frac{11}{10}$ $\begin{array}{r} 1 \\ 10\overline{)\,11} \\ \underline{10} \\ 1 \end{array}$ $1\frac{1}{10}$

   c. $12\frac{4}{5}$ $12 \times 5 = 60$ $60 + 4 = 64$ $\frac{64}{5}$

   d. $\frac{19}{15}$ $\begin{array}{r} 1 \\ 15\overline{)\,19} \\ \underline{15} \\ 4 \end{array}$ $1\frac{4}{15}$

   e. $7\frac{7}{8}$ $7 \times 8 = 56$ $56 + 7 = 63$ $\frac{63}{8}$

5. a. $\frac{18}{81} = \frac{\frac{18}{9}}{\frac{81}{9}} = \frac{2}{9}$

b. $\frac{9}{12} = \frac{\frac{9}{3}}{\frac{12}{3}} = \frac{3}{4}$

c. $\frac{21}{63} = \frac{\frac{21}{7}}{\frac{63}{7}} = \frac{3}{9} = \frac{\frac{3}{3}}{\frac{9}{3}} = \frac{1}{3}$

d. $\frac{3}{7}$ (in lowest terms)

e. $\frac{14}{21} = \frac{\frac{14}{7}}{\frac{21}{7}} = \frac{2}{3}$

f. $\frac{16}{64} = \frac{\frac{16}{4}}{\frac{64}{4}} = \frac{4}{16} = \frac{\frac{4}{4}}{\frac{16}{4}} = \frac{1}{4}$

6. a. $\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

b. $\frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21} = \frac{1}{21}$

c. $\frac{3}{8} + \frac{3}{4} = \frac{3}{8} + \frac{6}{8} = \frac{9}{8}$ or $1\frac{1}{8}$

d. $2\frac{3}{8} = 1\frac{11}{8}$

$-1\frac{5}{8} = -1\frac{5}{8}$

$\frac{6}{8} = \frac{3}{4}$

e. $6\frac{7}{10} = 6\frac{7}{10} = 5\frac{17}{10}$

$-4\frac{4}{5} = -4\frac{8}{10} = -4\frac{8}{10}$

$1\frac{9}{10}$

f. $11\frac{3}{4} + 19\frac{5}{8} + 9\frac{1}{2} + \frac{3}{16} =$

$11\frac{12}{16} + 19\frac{10}{16} + 9\frac{8}{16} + \frac{3}{16} = 39\frac{33}{16}$

$16\overline{)33}$ or $2\frac{1}{16}$ $\quad 39 + 2\frac{1}{16} = 41\frac{1}{16}$

$\frac{32}{1}$

7. a. $\frac{\cancel{2}^{1}}{1} \times \frac{\cancel{4}^{1}}{\cancel{3}} \times \frac{\cancel{2}}{\cancel{3}_{1}} = \frac{1}{4}$

b. $4\frac{2}{3} \times 5\frac{1}{4} \times 2\frac{2}{3} =$

$4 \times 3 = 12$ $\quad 5 \times 4 = 20$ $\quad 2 \times 3 = 6$

$12 + 2 = 14$ $\quad 20 + 1 = 21$ $\quad 6 + 2 = 8$

$\frac{14}{3} \times \frac{\cancel{21}^{7}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{2}_{1}} = \frac{196}{3}$

$3\overline{)196}$ or $65\frac{1}{3}$

$\frac{18}{16}$

$\frac{15}{1}$

c. $\frac{3}{4} \times 5\frac{1}{2} =$ $\quad 5 \times 2 = 10$ $\quad 10 + 1 = 11$

$\frac{3}{4} \times \frac{11}{2} = \frac{33}{8}$

$8\overline{)33}$ or $4\frac{1}{8}$

$\frac{32}{1}$

d. $\frac{1}{8} \times 16 = \frac{1}{\cancel{8}} \quad \frac{\overset{2}{\cancel{16}}}{1} = 2$
$\quad\quad\quad

LESSON 3

DECIMALS

CREDIT HOURS ---------- 1

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:

		Begin at FRAME
1.	Write, in his own words, the definition of a decimal.	3-1 (p.3-2)
2.	Demonstrate ability to read decimals by matching numerical decimals with the appropriate word decimals.	3-8 (p.3-16)
3.	Write the numerical form of given word decimals.	3-14 (p.3-28)
4.	Change fractions to decimals.	3-19 (p.3-6)
5.	Change decimals to fractions and reduce the fractions to lowest terms.	3-39 (p.3-15)
6.	Round off decimals.	3-21 (p.3-10)
7.	Add decimals.	3-44 (p.3-25)
8.	Subtract decimals.	3-49 (p.3-5)
9.	Multiply decimals.	3-56 (p.3-19)
10.	Divide decimals.	3-50 (p.3-7)

SET 1. DEFINITION OF DECIMAL

Frames 3-1 through 3-16 are at the top of even numbered pages.

FRAME 3-1

The definition of a decimal is: A number that represents a fraction with a denominator that is a power of ten. The definition of a decimal is: _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____.

(.0004) (3-16)

FRAME 3-17

Write the numerical form of twenty-nine thousandths.

_____.

FRAME 3-33

3/4 is not correct; .75 = 3/4. Return to page 3-15, frame 3-39 and work the problem again. Then select the correct answer and continue with the program.

FRAME 3-48

You set up your problem incorrectly and had the decimal in the wrong place. This is what you should have set up for your division: $2\overline{)1.00}$. Return to page 3-6, frame 19, and determine the correct answer. Then turn to the correct answer page.

(A number that represents a fraction with a denominator that is a power of ten) (3-1)

FRAME 3-2

Being a power of ten simply means that you can divide ten into the number evenly. The fraction $\frac{47}{100}$ has a denominator of one hundred and, of course, ten will divide evenly into it. We know that 100 is a power of _____.

(.029) (3-17)

FRAME 3-18

Write the numerical form of each of the following word decimals

a. Sixty-five hundredths _____

b. Sixty and ninety-seven thousandths _____

c. Three hundred and four tenths _____

d. Seventy-five ten thousandths _____

e. Fifty-eight and sixty-six hundredths _____

f. Forty-nine thousandths _____

FRAME 3-34

You have misplaced the decimal point. The decimal point ALWAYS goes to the extreme right of the dividend. EXAMPLE: $_\overline{)12}$. not $_\overline{)1.2}$, from $\frac{12}{7}$. Return to page 3-11, frame 3-37, rework the problem and continue with the program.

SET 8. SUBTRACTING DECIMALS

(36,96) (3-44)

FRAME 3-49

Right. The main thing to remember is to keep the decimal points lined up under each other. Now let's subtract decimals. The rules are the same as they are in the subtraction of whole numbers. Just as in the addition of decimals, the decimal points must be lined up under each other. You must also remember that the smaller of the numbers must go under the larger.

Solve this problem: 729.75308 - .0077 =

If your answer is:	Go to Frame:
729.75231	3-52 (p. 3-11)
729.74538	3-56 (p. 3-19)

(10 or ten) (3-2)

FRAME 3-3

All decimals represent fractions and in every case the denominator is a power of ten. The decimal, .1, represents the fraction $\frac{1}{10}$. The denominator, is a _____ of _____.

SET 4. CHANGING FRACTIONS TO DECIMALS

(a. .65; b. 60.097; c. 300.4; d. .0075; e. 58.66; f. .049) (3-18)

FRAME 3-19

All fractions can be changed to a decimal by dividing the numerator by the denominator. The decimal may be carried out as many places as the problem indicates. Example: $\frac{7}{8}$ to a decimal is $\begin{array}{r} .875 \\ 8\overline{)7.000} \end{array}$

BROKEN INTO STEPS:

a. Divide the numerator (7) by the denominator (8).

b. Place the decimal point to the right of the numerator.

c. Add zeros to the right of the decimal point as needed.

d. Place a decimal point in the quotient DIRECTLY over the decimal point in the division bracket.

e. Carry the quotient out as far as necessary.

Change $\frac{1}{2}$ to a decimal.

If your answer is:	Go to Frame:
2.0	3-43 (p. 3-23)
.5	3-30 (p. 3-28)
5.0	3-48 (p. 3-3)

FRAME 3-35

Your division is right, but it is unnecessary to put the 0 at the end of the decimal. Turn to page 3-28, frame 3-30 and continue the program.

SET 10. DIVIDING DECIMALS

(a. 66.42; b. .825) (3-59)

FRAME 3-50

Now let's divide decimals. The most important factor is that the divisor must be "made" a whole number before division is started. This is done by moving the decimal in the divisor all the way to the right. Example: $.25\overline{)\ \ }$ becomes $25.\overline{)\ \ }$. Then move the decimal in the dividend the same number of places to the right. Example: $.25\overline{)1.25}$ becomes $25.\overline{)125.}$ Move the decimal point in the following division problem and solve.

$$3.3\overline{).66}$$

NOTE: Go to page 3-27, frame 3-60.

(power; ten) (3-3)

FRAME 3-4

The definition of a decimal is: A number that represents a fraction with a _____ that is a _____ of _____.

FRAME 3-20

If you are reading this paragraph, then you are not following directions. From here on, you must follow the directions given in each frame VERY CAREFULLY. Return to the frame 3-19 and follow the directions given there.

FRAME 3-36

$\frac{45}{1000}$ is correct for the first step, but each fraction must be in its lowest terms. 5 divides into 45 and 1000--thus it can be reduced. Go back to page 3-21, frame 3-42 and reduce the fraction, choose the correct answer, and go to the page indicated.

$\left[3.2\overline{)\,.064}\right]$ (3-62)

FRAME 3-51

Right. REMEMBER: The <u>divisor</u> is to the <u>right</u> of the division sign. Solve these problems and <u>show your work</u>.

a. $4.9 \div .007 =$ b. $1179 \div 13.1 =$ c. $.02925 \div 2.25 =$

WORK HERE --

a. b. c.

Go to page 3-23, frame 3-58 for answers.

(denominator; power; ten) (3-4)

FRAME 3-5

Key words often help you remember hard to learn definitions. In the definition of DECIMAL, the words to remember as keys are: FRACTION, DENOMINATOR, AND POWER OF TEN.

Write the key words that will help you remember the definition of decimal.

_____, _____, and _____, _____, _____.

SET 6. ROUNDING OFF DECIMALS

[a. $\frac{7}{10}$; b. $\frac{9}{1000}$; c. $\frac{3}{4}$; d. $\frac{1}{5}$] (3-20)

FRAME 3-21

In many cases, a large cumbersome decimal is not necessary. In those cases where a smaller decimal will do, you may ROUND OFF the decimal. To make a large decimal smaller and easier to use without losing a great deal of accuracy, you will _____ _____ the large decimal.

(.52) (3-30)

FRAME 3-37

.52 is correct. You have been changing proper fractions to decimals, so now let's change an IMPROPER FRACTION to a decimal. It is done in the same manner, but NOW the answer will include a whole number.

For example: $\frac{3}{2}$ changed to a decimal is $2\overline{)3.0.}$ $\begin{array}{r}1.5\\ \\ 2\\ \hline 10\\ 10\end{array}$ As you can see, an improper fraction will become a whole number and a decimal (1.5).

Change $\frac{12}{7}$ to a decimal.

If your answer is:	Go to Frame:
.17	3-34 (p. 3-5)
1.7	3-46 (p. 3-29)

FRAME 3-52

Remember when you were told that decimal points must go under decimal points? Well, the error you made was because of the decimal placement. A good way to remember the decimal points is put <u>them</u> on the paper first (in a column) and <u>then</u> put the numbers down. Also remember to put the decimal in the answer DIRECTLY under those in the column. Go back to page 3-5, frame 3-49 and do the problem again.

(fraction; denominator; power of ten) (3-5)

FRAME 3-6

A decimal is a number that represents a _____ with a _____ that is _____ _____ _____ _____.

(round off) (3-21)

FRAME 3-22

Rounding off involves THREE steps. The FIRST TWO are:

a. Determine the PLACE you want to round off to. (Tenths, hundredths, etc.)

b. Then look at the number (digit) directly to the right of that place.

Example: .176

To round to hundredths: Look at the number to the right of the hundredths place. In this case, it is a 6. The FIRST number that you will look at when rounding .265 to TENTHS is _____.
number

FRAME 3-38

You have the 3 zeros but what happened to the 1? The decimal .679 is read "six hundred seventy-nine thousandths", so the denominator becomes 1000. Return to page 3-30, frame 3-31 and select the correct answer.

FRAME 3-53

No. DO NOT ADD an extra zero on the right of any answer. If you need zeros to make your digit count correct, they must go to the left of the answers. For example: .2 x .002 will equal .0004, not .4000.

Return to page 3-21, frame 3-57 and select the correct answer.

(fraction; denominator; a power of ten) (3-6)

FRAME 3-7

Write, in your own words, the definition of a decimal.

_____

_____

_____

(6) (3-22)

FRAME 3-23

You have the decimal .27364, and you want to round it off to tenths. What number would you look at first? (Circle your choice.)

a. 2 b. 7 c. 3 d. 6 e. 4

SET 5. CHANGING DECIMALS TO FRACTIONS

(Answers to problems in frame 3-46: a. .8; b. 5.2; c. .8; d. 1.3)

FRAME 3-39

You have learned how to change a fraction to a decimal, so let's change a decimal into a fraction. The FIRST thing to do is to make the digits of the decimal the NUMERATOR OF THE FRACTION. The denominator of the fraction will have a one (1), followed by the same number of zeros as there are digits in the decimal. For example, the decimal .27 becomes the fraction $\frac{27}{100}$. Notice how the digits 27 become the numerator and the denominator begins with a 1 and two zeros follow. There were two digits in the decimal, thus there are two zeros in the denominator. Change .7 to a fraction.

If your answer is:	Go to Frame:
$\frac{7}{100}$	3-31 (p. 3-30)
$\frac{7}{10}$	3-42 (p. 3-21)
$\frac{3}{4}$	3-33 (p. 3-3)

FRAME 3-54

There are more than two digits in the decimal .045. Zero IS a digit. That makes three digits in this decimal. You should use the same number of zeros as there are digits and make the denominator 1000.

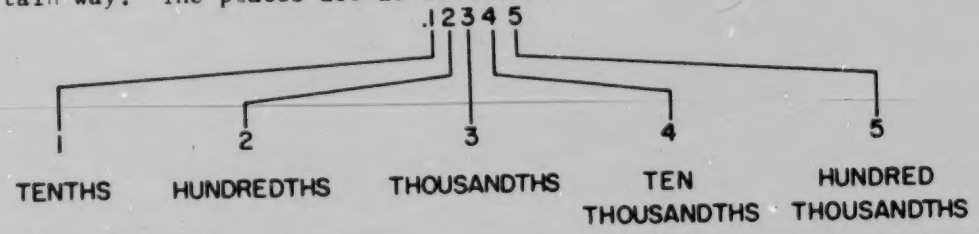
Return to page 3-21, frame 3-42 and select the correct answer.

SET 2. READING DECIMALS

(A number that represents a fraction with a denominator that is a power of ten (or similarly stated)) (3-7)

FRAME 3-8

Each digit in a decimal has a place value and is read in a certain way. The places are as follows:



The 3 in the thousandths place, the 5 is in the hundred thousandths place, and the 1 is in the _____ place.

---

(7) (3-23)

FRAME 3-24

The THIRD STEP is: If the number to the right of the place you are rounding off is 5 OR MORE, you ADD (+1) one to the place and drop the remainder of numbers.

For example: .176

This decimal rounded to tenths becomes .2 because the number to the right of the tenths place (7) is 5 or greater. Also, note that the 7 and 6 were dropped.

Round .0074 to the nearest HUNDREDTH. (Circle your answer.)

a. .01 b. .007 c. .1 d. .08

$\left[\frac{9}{200}\right]$ (3-42)

FRAME 3-40

Fine. As you have done on this problem, make sure that any fraction you are working with is in its lowest terms. Change the following decimals to fractions. Remember, REDUCE each to its lowest terms. If you still are not certain of just how to change decimals to fractions, go back to page 3-15, frame 3-39 and rapidly review. Change these to fractions:

a. .7000--

b. .009--

c. .75--

d. .2--

NOTE: Turn to page 3-10, frame 3-21 for answers.

FRAME 3-55

Your decimal point should have been placed like this:

$$\begin{array}{r} 3.217 \\ \times\ .471 \\ \hline 3217 \\ 22519 \\ 12868 \\ \hline 1.515207 \end{array}$$

If you had it any place else, return to page 3-19, frame 3-56 and read the rules again. If you did it correctly, do the following problems by placing the decimal points correctly in the product.

a.
$$\begin{array}{r} .0035 \\ \times\ 3.28 \\ \hline 280 \\ 70 \\ 105 \\ \hline 11480 \end{array}$$

b.
$$\begin{array}{r} 22.222 \\ \times\ .11 \\ \hline 22222 \\ 22222 \\ \hline 244442 \end{array}$$

NOTE: Go to page 3-21, frame 3-57

(tenths) (3-8)

FRAME 3-9

As you probably have noticed, the places to the right of the decimal point all end in "ths". In the decimal 2.46, the 6 is in the _____ place.

(.01) (3-24)

FRAME 3-25

When the number to the right is LESS THAN 5, do not change the place value and DROP THE REMAINDER OF THE NUMBERS.

Round the decimal .7848 to hundredths. (Circle your answer).

a. .78 b. .79 c. .785 d. .7800

FRAME 3-41

You neglected the decimal point. You must place decimal points DIRECTLY UNDER EACH OTHER. The sum will have the decimal point carried right down into it from the column being added. Return to page 3-25, frame 3-44 and do the problem again. Remember to put the decimal points under each other.

EXAMPLE:

```
  18.6
   .015
2056.11
+  1.1
-------
2075.825
```

SET 9. MULTIPLYING DECIMALS

(729.74538) (3-49)

FRAME 3-56

Right. You are now ready for multiplication. Decimals are multiplied just as whole numbers are, except you have a decimal point to put in the final answer (product).

Sample problem: .15 x 1.10 =

Place the larger number OVER the smaller and multiply just as you do in whole numbers. Ex.

```
 1.10
x .15
-----
 550
110
-----
1650
```

Count the number of digits to the right of the decimal points in the factors of the problem. Ex. 1.10 and .15 = 4 digits to the right in this case.

Count off 4 places FROM THE RIGHT in the PRODUCT, and place a decimal point. Ex. .1650 (product of this problem)

Place the DECIMAL POINT in the product of this problem:

```
 3.217
x .471
------
  3217
 22519
12868
-------
1515207
```

Turn to page 3-17, frame 3-55.

(Hundredths) (3-9)

FRAME 3-10

A decimal is read like this:

35.362 -- "Thirty-five AND three hundred sixty-two thousandTHS".

The 2 in this decimal is in the _____ place.

(.78) (3-25)

FRAME 3-26

REMEMBER:

a. FIRST, look at the number to the right of the place you are rounding off.

b. If the number is 5 or more, add 1 to the place.

c. If less than 5, do not add anything.

d. Always drop the remainder of digits to the right of the rounded off place.

Round these decimals to the indicated places:

Tenths: .408062 _____

Hundredths: .408062 _____

Thousandths: .408062 _____

Ten thousandths: .408062 _____

Hundred Thousandths: .408062 _____

$\left[\frac{7}{10}\right]$ (3-39)

FRAME 3-42

Very good. The next thing to remember is: Make sure the fraction is in its lowest terms. For example, changing the decimal .5 to a fraction, it first becomes $\frac{5}{10}$. Is this in the lowest terms possible? Of course, the answer is no. In its lowest terms, it would be $\frac{1}{2}$. Always check the fraction and be sure it is in its lowest terms. Try this one now. Change .045 to a fraction.

If your answer is:	Go to Frame:
$\frac{9}{200}$	3-40 (p. 3-17)
$\frac{45}{1000}$	3-36 (p. 3-9)
$\frac{45}{100}$	3-54 (p. 3-15)

(a. .011480 or .01148 b. 2.44442) (3-55)

FRAME 3-57

Let's try another, to make sure that you have the decimal point placement down pat. Solve this one: .55 x .003 =

If your answer is:	Go to Frame:
.01650	3-53 (p. 3-13)
.00165	3-59 (p. 3-25)

(thousandths) (3-10)

FRAME 3-11

When there is a whole number and a decimal, the decimal point is read "AND". For example: 6.02 is read "six AND two hundredths".

When there is only a decimal (no whole number), it is read without using the word, "and". For example: .06 is read "six hundredths".

How would "thirty-three THOUSANDTHS" be written as a decimal?

_____

(.4; .41; .408; .4081; .40806) (3-26)

FRAME 3-27

Round off the following decimals:

To Hundredths: 41.1145-- _____

.98509-- _____

To Tenths: .6419-- _____

To Ten Thousandths: .29826-- _____

1.11181-- _____

FRAME 3-43

In order to change a fraction to a decimal, you divide the numerator by the denominator. You did not do this. In the case of $\frac{1}{2}$, the denominator (2) is divided into the numerator (1) like this:

$$2\overline{)1.0}\quad .5$$
$$10$$

$\frac{1}{2}$ changed to a decimal is therefore .5. ALL fractions are changed to decimals in the same manner.

Change $\frac{3}{4}$ to a decimal.

If your answer is:	Go to frame:
.750	3-35 (p. 3-7)
.75	3-30 (p. 3-28)

(a. 700; b. 90; c. .013) (3-51)

FRAME 3-58

Solve these problems:

a. 289.0038 + .992763 =

b. .3928 - .02867 =

c. .42 x 3.7 =

d. 4.32 ÷ .0036 =

WORK ON SCRATCH PAPER.

Turn to page 3-29, frame 3-61

(.033) (3-11)

FRAME 3-12

REMEMBER-- When you are reading decimals, the decimal point is read "and" except when there is NO whole number. For example: .5 is read "five tenths". 3.22 is read "three AND twenty-two hundredths".

Match the decimal in column A with the correct word decimal in column B, placing the correct letter by the word decimal.

A		B
a. 4.3	_____	six hundreds
b. .006	_____	twenty-five and one hundredth
c. 25.01	_____	six hundredths
	_____	four and three tenths
	_____	twenty-five and one tenth
	_____	six thousandths

(Hundredths, 41.11, .99; Tenths, .6; Ten thousandths, .2983, 1.1118) (3-27)

FRAME 3-28

If you are NOT having trouble with rounding off, proceed to page 3-26, frame 3-29. If you are and your trouble is mainly knowing the "places", turn to page 3-16, frame 3-8, and review. If you do not understand how to round off, return to page 3-10, frame 3-21 and review.

When you have corrected your trouble, go to page 3-26, frame 3-29.

SET 7. ADDING DECIMALS

(a. .3; b. .1; c. .11; d. 8.34; e. .255; f. 5.974; g. 7.7778;

h. .0009; i. .09807; j. 3.00005) (3-29)

FRAME 3-44

Adding decimals is much the same as simple whole number addition. The difference is that there is a decimal point to keep in mind. The decimals are put in a column and decimal points are under decimal points (see example). The decimal point is brought down to the sum and the addition is carried on just as it is in whole number addition.

EXAMPLE:
```
  6.3
   .01
 22.22
 28.53
```

Add these decimals. 33.79 + .97 + 2.2 =

If your answer is:	Go to Frame:
36.96	3-49 (p. 3-5)
3498	3-41 (p. 3-19)

(.00165) (3-57)

FRAME 3-59

Very good. Care must be taken with your arithmetic. It is always a good idea to CHECK your multiplication and addition. This is where most of the errors are made, with a few being made on the placement of the decimal point.

Let's try two more. After completing them, check your arithmetic and decimal placement.

a. 332.1 x .2 =

b. .55 x 1.5 =

Turn to page 3-7, frame 3-50.

(a. 4.3 (four and three tenths) b. .006 (six thousandths) c. 25.01 (twenty-five and one hundredth) (3-12)

FRAME 3-13

Now, match column A with column B in the same manner.

A		B
a. .25	_____	Two hundredths
b. .002	_____	One and two hundred twenty-two thousandths
c. 20.05	_____	Twenty-five hundredths
d. 1.222	_____	Two thousandths
	_____	Twenty and five hundredths
	_____	Twenty-five hundred
	_____	One and two hundred twenty-two thousands

FRAME 3-29

Round off each of the following decimals to the indicated place.

To the nearest tenth:

a. .329-- _____

b. .05-- _____

To the nearest hundredth:

c. .10909-- _____

d. 8.3434-- _____

To the nearest thousandth:

e. .2551-- _____

f. 5.9738-- _____

To the nearest ten thousandth:

g. 7.777774-- _____

h. .000891-- _____

To the nearest hundred thousandth:

i. .0980653-- _____

j. 3.000051-- _____

NOTE: Go to page 3-25, frame 3-44

FRAME 3-45

Wrong. The number to the right of the division sign is always the divisor.

$.064 \div 3.2$ (3.2 is the divisor, not .064)

Return to page 3-31, frame 3-62 and select the correct answer.

$\left[33.\overline{)6.6}^{\;.2} \right]$ (3-50)

FRAME 3-60

$3.3\overline{).66}$ becomes $33.\overline{)6.6}^{\;.2}$ by moving the decimal point one place. When the divisor is a whole number and the dividend is a decimal, such as $33\overline{)6.4}$, you do not move the decimal point. Simply place the decimal point up in the quotient directly over the decimal point in the dividend; then divide. For example: $275\overline{)3.44}$

Solve this: $26\overline{)7.8}$

The answer to the problem above is: (Circle your answer).

a. 3 b. .3 c. .03

NOTE: Go to page 3-31, frame 3-62

SET 3. WRITING DECIMALS

(a. .25 (twenty-five hundredths); b. .002 (two thousandths); c. 20.05 (twenty and five hundredths); d. 1.222 (one and two hundred twenty-two thousandths)) (3-13)

FRAME 3-14

When writing a decimal, FIRST and MOST IMPORTANT, determine the "place" value (thousandths, tenths, etc.). This will give you the number of digits you need to the right of the decimal point. For example: twenty-two thousandths will require three digits because it is to the thousandths place. It would be written: .022 . Five and five tenths would be written: 5.5 (Remember, with a whole number, the decimal point is read AND.). How would twenty-five and four thousandths be written? _____

(.5) (3-19)

FRAME 3-30

Very good! You should be ready for a more difficult problem, so let's do this one:

Change $\frac{12}{23}$ to a decimal.

If your answer is:	Go to frame:
.0052	3-47 (p. 3-31)
.52	3-37 (p. 3-11)

FRAME 3-46

Right. If you want to review before you do the problems below, return to page 3-6, frame 3-19, read the rules, and then come back and solve the problems. If you think you are ready now, then change each of the fractions below to decimals.

a. $\frac{4}{5}$ b. $\frac{52}{10}$ c. $\frac{9}{11}$ d. $\frac{13}{10}$

NOTE: Answer on page 3-15, in frame 3-39

---

a. 289.003800 + .992763 = 289.996563

b. .39280 − .02867 = .36413

c. .42 × 3.7 = 294 + 126 = 1.554

d. $36\overline{)43200.} = 1200$ (36, 72, 72)

(3-58)

FRAME 2-61

If you missed any of the above problems, go to the part of the program that teaches that type of problem and read the rules again. THEN correct your error. The pages that teach each function are listed below.

ADDITION (Page 3-25, frame 3-44)

MULTIPLICATION (Page 3-19, frame 3-56)

SUBTRACTION (Page 3-5, frame 3-49)

DIVISION (Page 3-7, frame 3-50 and page 3-31, frame 3-62)

This completes your lesson in decimals. Working partial numbers is easier when you use decimals rather than fractions, so this lesson is very important. Go to the SELF-TEST on page 3-33 and test your understanding of decimals.

(25.004) (3-14)

FRAME 3-15

Thirteen and four tenths would appear as 13.4. Nine and forty-four hundredths appear as: _____

FRAME 3-31

Wrong!! You add only the number of zeros that there are digits in the decimal. There is only one digit in the decimal .7, so there will be only one zero in the fraction. The decimal .679 has three digits, so the denominator will have three zeros and the fraction would look like this _____.

If your answer is:	Go to Frame:
679/1000	3-42 (p. 3-21)
679/000	3-38 (p. 3-13)

FRAME 3-47

Wrong. You set your division up incorrectly. The problem should have been set up like this: $23\overline{)12.000}$

Return to page 3-28, frame 3-30 and do the division again and place the decimal point in the right position; then select the right answer and go to the page indicated.

(.3) (3-60)

FRAME 3-62

$26\overline{)7.8}$ solved is: $26\overline{)\overset{.3}{7.8}}$

If the dividend is a whole number, Ex: $1.32\overline{)25.}$, add zeros and move the decimal point. Ex: $1.32\overline{)25.00}$. When the decimal has been moved as appropriate, then place a decimal point in the quotient directly over the point in the dividend, Ex: $2.5\overline{).1.00}$ and solve.

Example $2.5\overline{)\overset{.04}{.1.00}}$

NOTICE HOW THE QUOTIENT IS .04 AND NOT .4. THIS IS BECAUSE 25 GOES INTO 10 ZERO TIMES, AND INTO 100 FOUR TIMES.

Solve the problem below:

$$.064 \div 3.2 =$$

NOTE: ÷ is the sign for division and the number on the <u>right is always</u> the divisor.

If your answer is:	Go to Frame:
$.064\overline{)\overset{50.}{3.200}}$	3-45 (p. 3-27)
$3.2\overline{)\overset{.2}{.064}}$	3-32 (p. 3-32)
$3.2\overline{)\overset{.02}{.064}}$	3-51 (p. 3-9)

(9.44) (3-15)

FRAME 3-16

Four ten thousandths looks like _____.

Turn back to the bottom of page 3-2, frame 3-17.

FRAME 3-32

No. Move the decimal point in the dividend the same number of places as you did in the divisor. Example: $3.2\overline{).064}$ becomes $32.\overline{)0.64}$ . .

Return to page 3-31, frame 3-62 and select the correct answer.

SELF-TEST ON DECIMALS

1. Write, in your own words, the definition of a decimal.

_____

_____

2. Match the numerical decimals in Column A with the appropriate word decimals in Column B. Place the letter from Column A in the blank next to the correct word decimal in Column B.

A	B
a. 30.04	_____ One hundred forty-six
b. .379	_____ Three hundred seventy-nine thousandths
c. 1.46	_____ Thirty and four hundredths
d. 90.001	_____ Ninety-one thousands
	_____ Three hundred seventy-nine thousand
	_____ One and forty-six hundredths
	_____ Thirty four hundredths
	_____ Ninety and one thousandths

3. Write the numerical form of the following word decimals:

a. Nine and seventy-five hundredths _____

b. Twelve and three tenths _____

c. Seven and one hundred twenty-three thousandths_____

d. Seventy-three ten thousandths _____

4. Change the fractions below to decimals.

a. $\frac{3}{10}$

b. $\frac{4}{5}$ =

c. $\frac{3}{4}$ =

d. $\frac{5}{2}$ =

5. Change the decimals below to fractions. REDUCE TO LOWEST TERMS.

a. .25 =

b. .9 =

c. .105 =

d. .35 =

6. Round off the following decimals as directed.

NEAREST TENTH:

a. .6354--

NEAREST HUNDREDTH:

b. 13.8467--

NEAREST THOUSANDTH:

c. .05671--

NEAREST TEN THOUSANDTH:

d. 1.60006--

7. Add the following decimals:

a. 9.37 + 15.756 + .76 =

b. 69.333 + .12 + 111.1 =

c. .0055 + 7.02 + 12.367 =

8. Subtract the following decimals:

   a. 13.14 - 2.96 =

   b. .7068 - .077 =

   c. 447.3 - .93 =

9. Multiply the following decimals:

   a. .03 x 10.31 =

   b. .71 x .004 =

   c. 1.51 x .212 =

10. Divide the following decimals:

   a. .08 ÷ .004 =

   b. .00344 ÷ 3.44 =

   c. .04 ÷ .08 =

ANSWERS TO SELF-TEST LESSON 3

DECIMALS

1. A decimal is a number that represents a fraction with a denominator that is a power of ten.

2. a. (30.04 thirty and four hundredths)

   b. (.379 three hundred seventy-nine thousandths)

   c. (1.46 one and forty-six hundredths)

   d. (90.001 ninety and one thousandths)

3. a. 9.75 b. 12.3 c. 7.123 d. .0073

4. a. $\frac{3}{10}$

$$10\overline{)3.0}\quad .3,\ 3\,0,\ 0$$

   b. $\frac{4}{5}$

$$5\overline{)4.0}\quad .8,\ 4\,0,\ 0$$

   c. $\frac{3}{4}$

$$4\overline{)3.00}\quad .75,\ 2\,8,\ 20,\ 20,\ 0$$

   d. $\frac{5}{2}$

$$2\overline{)5.0}\quad 2.5,\ 4,\ 10,\ 10,\ 0$$

5. a. $.25 = \frac{25}{100} = \frac{\frac{25}{5}}{\frac{100}{5}} = \frac{5}{20} = \frac{\frac{5}{5}}{\frac{20}{5}} = \frac{1}{4}$

   b. $.9 = \frac{9}{10}$

   c. $.105 = \frac{105}{1000} = \frac{\frac{105}{5}}{\frac{1000}{5}} = \frac{21}{200}$

   d. $.35 = \frac{35}{100} = \frac{\frac{35}{5}}{\frac{100}{5}} = \frac{7}{20}$

6. a. .6 In .6354, the number to the right of the tenth's place (3) is less than 5.

b. 13.85 In 13.8467- , the number to the right of the hundred's place (6) is greater than 5.

c. .057 In .05671- , the number to the right of the thousandth's place (7) is greater than 5.

d. 1.6001 In 1.60006- , the number to the right of the ten thousandth's place (6) is greater than 5.

7. a. $\begin{array}{r} 9.37 \\ 15.756 \\ +\ .76 \\ \hline 25.886 \end{array}$ b. $\begin{array}{r} 69.333 \\ .12 \\ +\ 111.1 \\ \hline 180.553 \end{array}$ c. $\begin{array}{r} .0055 \\ 7.02 \\ +\ 12.367 \\ \hline 19.3925 \end{array}$

8. a. $\begin{array}{r} 13.14 \\ -\ 2.96 \\ \hline 10.18 \end{array}$ b. $\begin{array}{r} .7068 \\ -.077 \\ \hline .6298 \end{array}$ c. $\begin{array}{r} 447.3 \\ -\ .93 \\ \hline 446.37 \end{array}$

9. a. $\begin{array}{r} 10.31 \\ \times\ .03 \\ \hline .3093 \end{array}$ b. $\begin{array}{r} .71 \\ \times\ .004 \\ \hline .00284 \end{array}$ c. $\begin{array}{r} 1.51 \\ \times\ .212 \\ \hline 302 \\ 151 \\ 302 \\ \hline .32012 \end{array}$

10. a. $.004\overline{)\begin{array}{r} 20.0 \\ .0800 \\ 8 \\ \hline 000 \end{array}}$ b. $3.44\overline{)\begin{array}{r} .001 \\ .00344 \end{array}}$ c. $.08\overline{)\begin{array}{r} .5 \\ .040 \\ 40 \\ \hline 0 \end{array}}$

LESSON 4

UNIT CONVERSION

CREDIT HOURS ---------- 1

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Write the definition of measure and unit of measure.	4-1 (p.4-2)
2. List the three fundamental units of measure.	4-4 (p.4-8)
3. List the two major measurement systems in use today.	4-7 (p.4-14)
4. Convert a measurement from one unit of measure to another.	4-12 (p.4-2)
5. Select from a list of measurements those that have simple units and those that have complex units.	4-23 (p.4-3)
6. Change a complex unit to a simple unit.	4-25 (p.4-7)
7. Change a simple unit to a complex unit.	4-36 (p.4-7)
8. Solve problems where measurements are used in the arithmetic operation.	4-38 (p.4-11)

SET 1. DEFINITION OF MEASURE AND UNIT OF MEASURE

Frames 4-1 through 4-11 are at the top of even numbered pages.

FRAME 4-1

Measure is the determination of the extent, dimension, capacity or amount of a quantity. Measurements are a fundamental part of all military engineering operations. Engineers are constantly determining what is the size of an object or how much of an object is required to do a job and these all involve a _____ of a quantity.

SET 4. CONVERTING UNITS OF MEASURE

Frames 4-12 through 4-22 are on the bottom of even numbered pages.

(sq ft or ft^2; g)

FRAME 4-12

The unit in which a measurement is expressed will not always be the unit in which you want to work. For example, you may know that you want to put up 2.3 miles of fence. However, you must order the fence by the foot. To solve this problem you would want to convert the miles to _____.

SET 5. SIMPLE AND COMPLEX UNITS

Frames 4-23 through 4-33 are at the top of odd numbered pages.

$$\left[7.5\frac{Km}{min} \times \frac{1000m}{1\ Km} \times \frac{1\ min}{60sec} = 125\frac{m}{sec}\right] \quad (4\text{-}22)$$

FRAME 4-23

All measurements discussed thus far have had simple units. A simple unit has only one length, mass, or time unit of measure, but may be any combination of these. For example, feet (length), second (time) or feet per second (length per time) are examples of simple units. Based on the above definition and example, does 2 minutes 15 seconds have simple units?

Circle One:

a. Yes b. No c. Under certain conditions

Frames 4-34 through 4-44 are on the bottom of odd numbered pages.

$$\left[2hr\ 45min\ 15sec = 2hr \times \frac{60min}{hr}\ \ 45min\ 15sec\right.$$

$$120min\ 45min\ 15sec = 165min\ 15sec =$$

$$\left.165min \times \frac{60sec}{min}\ \ 15sec = 9900sec\ \ 15sec = 9915sec\right] \quad (4\text{-}33)$$

FRAME 4-34

Now let's try a complex measurement in the Metric System.

3 hectograms 5 dekagrams 7 grams will reduce to _____ hectograms.

(measure) (4-1)

FRAME 4-2

The magnitude or size of an object consist of two parts, a number and a unit. The unit is called the unit of measure. The unit of measure is a known and accepted standard that has been defined by custom or by law. The standard that is used to describe a quantity's extent, dimension, capacity or amount is called _____ _____ _____.

(feet) (4-12)

FRAME 4-13

It may be necessary to convert from one system to another. This is particularly true when you are trading or buying from a foreign country. Panel 4-3, page 4-28 gives the English and Metric Systems equivalents. The equivalent of 1 cu ft in the English System is _____cubic meters in the Metric System.

(No, because two time units are used together and simple units can only have one time unit.) (4-23)

FRAME 4-24

Complex units exists when two or more length, mass, or weight, and time units are used together, to describe measurement. For example, 3 yards, 2 feet, 6 inches, has complex units because it uses three units of length to describe the total length. Which of the following have complex units?

a. 5lb 4oz b. 2240 $\frac{\text{lb}}{\text{long ton}}$ c. 24hr d. 2gal 1pt

---

$\left[3\text{Hg}\ 5\text{Dg}\ 7\text{g} = 3\text{Hg}\ 5\text{Dg} \times \frac{10\text{g}}{\text{Dg}}\ 7\text{g} = 3\text{Hg}\ 50\text{g}\ 7\text{g} = 3\text{Hg}\ 57\text{g}\right.$

$\left. = 3\text{Hg}\ 57\text{g} \times \frac{\text{Dg}}{10\text{g}} \times \frac{\text{Hg}}{10\text{Dg}} = 3\text{Hg}\ .57\text{Hg} = 3.57\text{Hg}\right]$ (4-34)

FRAME 4-35

In some problems, the intermediate units are not required to express the measurement. For example, the measurement 5 gallons 1 pint does not use the intermediate unit, quarts. The same steps in reducing are used, only two or more conversion factors may be required in some step. The above example reduces to _____ pints.

(unit of measure) (4-2)

FRAME 4-3

Write in your own words the definition of (1) measure and (2) unit of measure.

(1) _____

_____

(2) _____

_____

_____

(.0283) (4-13)

FRAME 4-14

There are three steps in converting a measurement from one unit to another. The first step is to determine the relationship between the units in an equivalence table (Panels 4-1 and 4-2) or conversion table (Panel 4-3). If you were converting meters to feet, you would select 1 meter = _____.

SET 6. CHANGE COMPLEX UNITS TO SIMPLE UNITS

(a; d) (4-24)

FRAME 4-25

It is better to reduce complex units to a simple unit before performing any arithmetic operation. A complex unit can be reduced to any one of the units which make it up. It is more common to reduce the complex units to the largest or smallest unit. If 3 yards 2 feet 6 inches is to be reduced to the smallest unit, it would be reduced to _____.
(unit of measure)

SET 7. CHANGE SIMPLE UNITS TO COMPLEX UNITS

$\left[5\text{gal } 1\text{pt} = 5\text{gal} \times \frac{4\text{qt}}{\text{gal}} \times \frac{2\text{pt}}{\text{qt}} \quad 1\text{pt} = 40\text{pt} \quad 1\text{pt} = 41\text{pt}\right]$ (4-35)

FRAME 4-36

It is often desirable to express a number with simple units in complex units. If the simple unit is small, this is done as follows:

Convert 7250sec to hr min sec

$$\frac{7250\text{sec}}{\frac{60\text{sec}}{\text{min}}} = 120\text{min } 50\text{sec}$$

$$60\frac{\text{sec}}{\text{min}} \overline{)\,7250\text{sec}} \quad 120\text{min}$$
$$\underline{60}$$
$$125$$
$$\underline{120}$$
$$50\text{sec}$$

$$\frac{120\text{min}}{\frac{60\text{min}}{\text{hr}}}\ 50\text{sec} = 2\text{hr } 0\text{min } 50\text{sec}$$

$$60\frac{\text{min}}{\text{hr}} \overline{)\,120} \quad 2\text{hr}$$
$$\underline{120}$$
$$0\text{min}$$

24775 feet is equivalent to _____mi_____yd_____ft.

SET 2. UNITS OF MEASURE

(1. measure is the determination of the extent, dimension, capacity or amount of a quantity; 2. unit of measure is a known and accepted standard that has been defined by custom or law that is used to describe quantity's extent, dimension, capacity or amount) (4-3)

FRAME 4-4

The number in the expression of the magnitude or size of an object indicates how many of the units of measure are required to be equal to the magnitude or size of the object. For example, in the expression 3 feet, the number 3 indicates the number of units of measure, feet, that are required to describe the distance between two points. The magnitude or size of an object consist of two parts, a _____ and a _____ _____ _____.

_____

(1 meter = 3.281 feet) (4-14)

FRAME 4-15

The second step is to divide both sides of the equivalence statement by the side that has the units you are converting. The results is known as the conversion factor. For example, if you were converting 2.5 meter to feet, you would

$$\frac{1 \text{ meter}}{1 \text{ meter}} = \frac{3.281 \text{ feet}}{1 \text{ meter}}$$

$$1 = \frac{3.281 \text{ feet}}{1 \text{ meter}} \qquad \text{(conversion factor)}$$

The units on the left side of the equal sign (meter) cancel out, resulting in 1. If you were converting feet to meters, you would divide both sides of the equivalence statement by _____.

(inches) (4-25)

FRAME 4-26

First, let's consider the problem of reducing 3 yards 2 feet 6 inches to the smallest given unit or inches. First step is to multiply the given number of the largest unit by the conversion factor that will reduce it to the next smaller given unit. In this problem, yards is the largest unit and feet is the next smaller unit. So, 3 yards will reduce to _____feet.

(4mi 1218yd 1ft) (4-36)

FRAME 4-37

If the simple unit is large, the measurement can be converted to a complex unit as follows: Convert $7\frac{1}{2}$ yd to yd ft in.

$$7\text{yd} + \left[\frac{1}{2}\text{yd} \times \frac{3\text{ft}}{\text{yd}}\right] = 7\text{yd} + \frac{3}{2}\text{ ft} = 7\text{yd } 1\frac{1}{2}\text{ ft} =$$

$$7\text{yd } 1\text{ft} + \left[\frac{1}{2}\text{ft} \times \frac{12\text{in}}{\text{ft}}\right] = 7\text{yd } 1\text{ft} + \frac{12\text{in}}{2} = 7\text{yd } 1\text{ft } 6\text{in}$$

$22\frac{7}{8}$ gallons is equivalent to _____gal_____qt_____pt.

(number; unit of measure) (4-4)

FRAME 4-5

There are three fundamental units of measure; length, mass, and time. The unit of measure, length, describes the space occupied by an object. The unit of measure, mass, describes the quantity of matter contained in a body. The unit of measure, time, is based on the movements of the heavenly bodies. The distance between two points would be described by the unit of measure _____.

(1 foot) (4-14

FRAME 4-16

The third step in converting a measurement from one unit to another is to multiply the known measurement by the number developed in the second step and cancel the unwanted units. For example, to convert the 2.5 meters to feet

$$2.5 \text{ meters} \times \frac{3.281 \text{ feet}}{1 \text{ meter}} = _____$$

(9) (4-26)

FRAME 4-27

The measurement now looks like this: 9 feet 2 feet 6 inches. The next step is add the product of the previous step to the given number of that next smaller unit. In this problem, the sum of this step is _____.

SET 8. SOLUTION OF PROBLEMS WHERE MEASUREMENTS ARE USED

(22gal 3qt 1pt) (4-37)

FRAME 4-38

As stated previously it is better to reduce complex units to a simple unit before performing any arithmetic operation. Reduce 3 feet 5 inches and 1 foot 9 inches to inches.

(length) (4-5)

FRAME 4-6

The three fundamental units of measure are _____,

_____and _____.

(8,2025) (4-16)

FRAME 4-17

Let's try another problem. In 5000 cubic inches there are

_____gallons.

(11 feet) (4-27)

FRAME 4-28

The third step is to repeat the processes of the first two steps until the given number of the smallest unit has been added. The final sum is the desired result. Complete the problem of reducing 3 yards 2 feet 6 inches to the smallest given unit. You now have 11 feet 6 inches. Now convert 11 feet to inches and add the 6 inches. You will then have the problem reduced to inches.

(3 feet 5 inches = 41 inches and 1 foot 9 inches = 21 inches) (4-38)

FRAME 4-39

When numbers with units are added or subtracted, the units must be the same before the operation is performed and the unit of the sum or difference will be the same. Find the sum of and the difference between 3 feet 5 inches and 1 foot 9 inches in inches.

a. Sum _____

b. Difference _____

SET 3. TWO MAJOR MEASUREMENT SYSTEMS

(length, mass, time) (4-6)

FRAME 4-7

Throughout history, countries have established their own distinct standards of measure. At the present time all the leading civilized countries except Russia, United States, and British Empire use the same system of measure. The system in common use in the United States and the British Empire is called the English System. The system used elsewhere except Russia is the Metric System. The most commonly used systems of measure are _____and_____.

$$\left[5000 \text{ cu in} \times \frac{1 \text{gal}}{231 \text{ cu in}} = 21.65 \text{gal}\right] \quad (4\text{-}17)$$

FRAME 4-18

Let's try another problem. If the required equivalence statement is not in the table, you must use two or more of the equivalence statements. For example, in 3 square yards there are _____square inches.

(138) (4-28)

FRAME 4-29

It is often desirable to reduce a measurement to the largest unit. To do this, begin with the next unit smaller than the desired unit, reduce this and all smaller units to the smallest given unit by the steps already discussed. If the measurement were 3 yards 2 feet 6 inches, the results of this first step would be_____.

(a. 62 inches; b. 20 inches) (4-39)

FRAME 4-40

When numbers with units are multiplied, both the number and the units are multiplied. The product of 2 feet and 3 feet is 2 x 3 (ft)(ft) or 6 sq ft. The number 2 and 3 are multiplied together and the units feet and feet are multiplied together. What is the product of 3 feet 6 inches and 1 foot 9 inches in square inches?

FRAME 4-8

The standard unit of length in the English system is the yard, standard unit of mass (or weight) is the pound, and standard unit of time is the second. Panel 4-1 (p. 4-24) gives the equivalents of the other units in the English system to the standards and to each other. After studying the panel, determine that the equivalent of 1 square foot is _____ square inches.

$$\left[3 \text{ sq yd} \times \frac{9 \text{ sq ft}}{1 \text{ sq yd}} \times \frac{144 \text{ sq in}}{1 \text{ sq ft}} = 3888 \text{ sq in}\right] \quad (4\text{-}18)$$

FRAME 4-19

Now try this problem. How many millimeters (mm) are there in 2.5 meters?

(2 feet 6 inches = 30 inches) (4-29)

FRAME 4-30

The next step is to use the results just obtained as a numerator and write a fraction whose denominator is the number of the smallest given units contained in the largest desired unit. This fraction should be reduced to its lowest terms. In the present problem, this fraction would be _____.

(42 inches x 21 inches = 882 sq in) (4-40)

FRAME 4-41

When numbers with units are divided, both the number and the units are divided. If the units are the same for both measurements, then they cancel. For example, if 9 feet is divided by 3 feet, the answer is 3. The feet in the denominator divides into or cancels the feet in the numerator. What is the quotient of 3 feet 6 inches divided by 1 foot 9 inches?

(144) (4-8)

FRAME 4-9

In the Metric system, the standard unit of length is the meter, standard unit of mass is the kilogram, and the standard unit of time, as in the English system, is the second. Panel 4-2 (p. 4-26) gives the equivalents of the other units in the metric system to the standards and to each other. Study the panel carefully, and determine that the equivalent of 1 meter is _____ decimeters.

$$\left[2.5\text{m} \times \frac{10\text{dm}}{\text{m}} \times \frac{10\text{cm}}{\text{dm}} \times \frac{10\text{mm}}{\text{cm}} = 2500\text{mm}\right] \quad (4\text{-}19)$$

FRAME 4-20

You receive orders to repair a bridge that is at a distance of 29.3 kilometers along a given route. Since you are not familiar with the route, you want to use the truck odometer to measure the distance in miles so that you will know when you are nearing the bridge. How many miles should you travel?

$\left[\frac{30\text{in}}{36\text{in/yd}} = \frac{5}{6}\text{ yd}\right]$ (4-30)

FRAME 4-31

The final step is to add this fraction to the given number of the largest unit. The resulting mixed number is the desired result. This mixed number in the present problem would be _____.

(42 inches ÷ 21 inches = 2) (4-41)

FRAME 4-42

What is the sum of 9m 3dm 7cm and 3m 2dm in cm?

(10) (4-9)

FRAME 4-10

If the numbers in panel 4-1 (English System) are compared with the numbers in panel 4-2 (Metric System), it will be noted the numbers in the metric system are much more uniform than those in the English system. The metric system is a decimal system, that is, the system is based on the number 10. All the numbers in panel 4-2 are multiples and are subdivisions of the number _____.

$\left[29.3 \text{ kilometers} \times \frac{.6215 \text{ mile}}{1 \text{ kilometer}} = 18.21 \text{ miles}\right]$ (4-20)

FRAME 4-21

All measurements discussed thus far have had only one unit, but some measurements have two or more units. For example, you think of a car as traveling at so many miles per hour or of concrete weighing so many pounds per cubic feet. When more than one unit exist, you convert each of the units to the desired units by multiplying by the required conversion factors.

Change $120\frac{\text{lb}}{\text{cu ft}}$ to pounds per cubic yard.

$\left[3\frac{5}{6} \text{ yards}\right]$ (4-31)

FRAME 4-32

A measurement can be reduced to an intermediate unit by first combining the units smaller than the desired intermediate unit with the intermediate unit by the steps just described and then reducing the units larger than intermediate unit by the steps previously described. For example, reduce 3 yards 2 feet 6 inches to feet.

$3\text{yd } 2\text{ft } \frac{6\text{in}}{12\text{in/ft}}$

$3\text{yd } 2\text{ft } \frac{1}{2}\text{ ft}$

$3\text{yd } 2\frac{1}{2}\text{ ft}$

$3\text{yd} \times 3\text{ft/yd} + 2\frac{1}{2}\text{ ft}$

$9\text{ft} + 2\frac{1}{2}\text{ ft} = _____$

$\left[9\text{m } 3\text{dm } 7\text{cm} = 9\text{m} \times \frac{10\text{dm}}{\text{m}} \times \frac{10\text{cm}}{\text{dm}} + 3\text{dm} \times \frac{10\text{cm}}{\text{dm}} + 7\text{cm} = 900\text{cm} + 30\text{cm} + 7\text{cm} = 937\text{cm}\text{; and } 3\text{m} \times \frac{10\text{dm}}{\text{m}} \times \frac{10\text{cm}}{\text{dm}} + 2\text{dm} \times \frac{10\text{cm}}{\text{dm}} = 300\text{cm} - 20\text{cm} = 320\text{cm}\text{;}\right.$

$\left.937\text{cm} + 320\text{cm} = 1257\text{cm}\right]$ (4-42)

FRAME 4-43

What is the product of 88ft/sec times 50min in miles?

(10) (4-10)

FRAME 4-11

Many of the words for the unit measure are long and it is common to abbreviate them. The abbreviation for each unit of measure is given after each unit in panels 4-1 and 4-2. You should become fa iar with these abbreviations. The abbreviation for square foot is _____ and for gram is _____.

Turn back to the bottom of page 4-2.

$$\left[120 \ \frac{\text{lb}}{\text{cu ft}} \times \frac{27 \text{ cu ft}}{1 \text{ cu yd}} = 3240 \ \frac{\text{lb}}{\text{cu yd}}\right] \quad (4\text{-}21)$$

FRAME 4-22

Let's try another problem involving two or more units. A rate of travel at 7.5 kilometers per minute (km/min) is how many meters per second?

Turn back to the top of page 4-3.

$\left[11\frac{1}{2}\right]$ (4-32)

FRAME 4-33

Let's try another problem of reducing a measurement with complex units to a simple unit. A trip required 2 hours 45 minutes 15 seconds. What is the time of the trip in seconds?

Turn back to the bottom of page 4-3.

$\left[50\text{min} = 50\text{min} \times 60\frac{\text{sec}}{\text{min}} = 3000\text{sec} \;;\; 88\frac{\text{ft}}{\text{sec}} \times 3000\text{sec} = 264000\text{ft};\right.$

$\left.264000\text{ft} \times \frac{1\text{ mi}}{5280\text{ft}} = 50\text{mi}\right]$ (4-43)

FRAME 4-44

You have completed the programmed text on measurements. Turn to the SELF-TEST and answer the questions. After checking the solution, review any of the material that gave you trouble.

PANEL 4-1

ENGLISH SYSTEM OF MEASURE

Measures of length:

12 inches (in)	= 1 foot (ft)
3 feet (ft)	= 1 yard (yd)
5½ yards (yd)	= 1 rod (rd)
320 rods	= 1 mile (mi)
5280 feet (ft)	= 1 mile (mi)
1760 yards (yd)	= 1 mile (mi)
8 furlongs	= 1 mile (mi)
1.15 miles	= nautical mile (naut mi)

Measure of area:

144 square inches (sq in or in*)	= 1 square foot (sq ft or ft^2)
9 square feet (sq ft or ft^2)	= 1 square yard (sq yd or yd^2)
30¼ square yards (sq yd or yd^2)	= 1 square rod (sq rd or rd^2)
160 square rods (sq rd or rd^2)	= 1 acre (A)
640 acres (ac)	= 1 square mile (sq mi or mi^2)

Measure of volume:

1728 cubic inches (cu in or in^3)	= 1 cubic foot (cu ft or ft^3)
27 cubic feet (cu ft or ft^3)	= 1 cubic yard (cu yd or yd^3)
128 cubic feet (cu ft or ft^3)	= 1 cord (cd)

Liquid Measures:

2 pints (pt)	= 1 quart (qt)
4 quarts (qt)	= 1 gallon (gal)
31½ gallons (gal)	= 1 barrel (bbl)
231 cubic inches (cu in or in^3)	= 1 gallon (gal)

Dry Measure:

2 pints (pt)	= 1 quart (qt)
8 quarts (qt)	= 1 peck (pk)
4 pecks (pk)	= 1 bushel (bu)
2150.42 cubic inches (cu in or in^3)	= 1 bushel (bu)

It should be noted that dry and liquid measures are different. For instance, 4 quarts in liquid measure contain 231 cu in, while in dry measure they contain 268.8 cu in nearly.

*Units expressed in this form are discussed in Lesson 7.

Measures of weight:

7000 grains (gr)	= 1 pound (lb)
16 ounces (oz)	= 1 pound (lb)
100 pounds (lb)	= 1 hundred weight (cwt)
2000 pounds (lb)	= 1 ton (T)
2240 pounds (lb)	= 1 long ton

Measure of time:

60 seconds (sec)	= 1 minute (min)
60 minutes (min)	= 1 hour (hr)
24 hours (hr)	= 1 day
365 days	= 1 common year (yr)
366 days	= 1 leap year.

PANEL 4-2

METRIC SYSTEM OF MEASURE

Measures of length:

10 millimeters (mm)	= 1 centimeter (cm)	.01 meter (m)
10 centimeters (cm)	= 1 decimeter (dm)	.1 meter
10 decimeters (dm)	= 1 meter (m)	1.0 meter
10 meters (m)	= 1 dekameter (Dm)	10 meters
10 dekameters (Dm)	= 1 hectometer (Hm)	100 meters
10 hectometers (Hm)	= 1 kilometer (Km)	1000 meters
10 kilometers (Km)	= 1 myriameter (Mm)	10000 meters

Measures of surface:

100 square millimeters (sq mm or mm^2)*	= 1 square centimeter (sq cm or cm^2)
100 square centimeters (sq cm or cm^2)	= 1 square decimeter (sq dm or dm^2)
100 square decimeters (sq dm or dm^2)	= 1 square meter (sq m or m^2) = 1 centare (ca)
100 square meters (sq m or m^2)	= 1 square dekameter (sq Dm or Dm^2) = 1 are (a)
100 square dekameters (sq Dm or Dm^2)	= 1 square hectometer (sq Hm or Hm^2) = 1 hectare (Ha)
100 square hectometers (sq Hm or Hm^2)	= 1 square kilometer (sq Km or Km^2)

Measure of volume:

1000 cubic millimeters (cu mm or mm^3)	= 1 cubic centimeter (cu cm or cm^3) or (cc)
1000 cubic centimeters	= 1 cubic decimeter (cu dm or dm^3) = 1 liter (l)
1000 cubic decimeters	= 1 cubic meter (cu m or m^3) = 1 kiloliter (Kl)

Measure of capacity:

10 milliliters (ml)	= 1 centiliter (cl)
10 centiliters	= 1 deciliter (dl)
10 deciliters	= 1 liter (l) = 1 cu dm
10 liters	= 1 dekaliter (Dl)
10 dekaliters	= 1 hectoliter (Hl)
10 hectoliters	= 1 kiloliter (Kl) = 1 cu m

*Units expressed in this form are discussed in Lesson 7.

Measures of weight:

10 milligrams (mg)	= 1 centigrams (cg)
10 centigrams (cg)	= 1 decigram (dg)
10 decigrams (dg)	= 1 gram (g)
10 grams (g)	= 1 dekagram (Dg)
10 dekagrams (Dg)	= 1 hectogram (Hg)
10 hectograms (Hg)	= 1 kilogram or kilo (Kg)
10 kilograms (Kg)	= 1 myriagram (Mg)
10 myriagrams (Mg)	= 1 quintal (Q)
10 quintals (Q)	= 1 millier, or metric ton (T)

Measures of time:

60 seconds (sec)	= 1 minute (min)
60 minutes (min)	= 1 hour (hr)
24 hours (hr)	= 1 day (da)
365 days (da)	= 1 common year (yr)
366 days (da)	= 1 leap year

PANEL 4-3

ENGLISH AND METRIC EQUIVALENTS

Unit	Equivalent	Unit	Equivalent
1 acre	.4047 hektare	1 mm	.03937 in
1 bushel	35.24 l	1 av oz	28.35 g
1 cm	.3937 in	1 Troy oz	31.10 g
1 cu cm	.0610 cu in	1 peck	8.809 l
1 cu ft	.0283 cu m	1 liq pt	.4732 l
1 cu in	16.387 cu cm	1 pound	.4536 Kg
1 cu m	1.308 cu yd	1 dry qt	1.101 l
1 cu m	35.31 cu ft	1 liq qt	.9464 l
1 cu yd	.7645 cu m	1 sq cm	.1550 sq in
1 foot	30.48 cm	1 sq ft	.0929 sq m
1 gallon	3.785 l	1 sq in	6.452 sq cm
1 grain av	.0648 g	1 sq mi	259 hektares
1 gram	15.43 gr av	1 sq m	1.196 sq yd
1 hektare	2.471 A	1 sq m	10.76 sq ft
1 inch	2.54 cm	1 sq yd	25.293 sq m
1 kilogram	2.205 lb	1 sq yd	.8361 sq m
1 kilometer	.6215 mile	1 ton	.9072 met ton
1 liter	.9081 dry qt	1 long ton	1.017 met ton
1 liter	1.057 liq qt	1 met ton	1.102 tons
1 meter	3.281 feet	1 met ton	.9842 long ton
1 mile	1.6093 Km	1 yard	.9144 meter

SELF-TEST ON MEASUREMENTS

1. Write the definition for measure.

_____

_____

2. Write the definition for unit of measure.

_____

_____

3. List the three fundamental units of measure.

   a. _____ b. _____ c. _____

4. List the two major measurement systems in use today.

   a. _____ b. _____

5. Change 5040 sq in to sq ft.

6. Change 2.35 Km to meters.

7. After each of the measurements in the following list, place a S if the units are simple and a C if the units are complex.

   a. 88ft/sec _____ d. 500min _____

   b. 1gal 3qt _____ e. 2Km 500m _____

   c. 5ft-lb/sq sec _____ f. 3Dg 3g _____

8. How many yards is 5yd, 1ft, 10in?

9. Convert 500 inches to yd ft in.

10. What is the product of 9 feet 3 inches times 2 feet 9 inches in square feet?

ANSWERS TO SELF-TEST LESSON 4

1. Measure is the determination of the extent, dimension, capacity or amount of a quantity.

2. Unit of measure is a known and accepted standard that has been defined by custom or law that is used to describe a quantity's extent, dimension, capacity or amount.

3. a. length b. mass or weight c. time

4. a. English b. Metric

5. 1 sq ft = 144 sq in

$$1 = \frac{1 \text{ sq ft}}{144 \text{ sq in}}$$

$$5040 \text{ sq in} \times \frac{1 \text{ sq ft}}{144 \text{ sq in}} = 35 \text{ sq ft}$$

6. $1\text{Km} = 10\text{Hm} \qquad 1 = \frac{10\text{Hm}}{\text{Km}}$

$1\text{Hm} = 10\text{Dm} \qquad 1 = \frac{10\text{Dm}}{\text{Hm}}$

$1\text{Dm} = 10\text{m} \qquad 1 = \frac{10\text{m}}{\text{Dm}}$

$$2.35\text{Km} \times \frac{10\text{Hm}}{\text{Km}} \times \frac{10\text{Dm}}{\text{Hm}} \times \frac{10\text{m}}{\text{Dm}} = 2350\text{m}$$

7. a. S d. S

b. C e. C

c. S f. C

8. $5\text{yd } 1\text{ft } 10\text{in} = 5\text{yd} + 1\text{ft} \times \frac{12\text{in}}{\text{ft}} \ 10\text{in}$

$5\text{yd } 12\text{in } 10\text{in} = 5\text{yd} + 22\text{in}$

$$5\text{yd } \frac{22\text{in}}{36\frac{\text{in}}{\text{yd}}} = 5\text{yd} - \frac{11}{18}\text{yd} = 5\frac{11}{18}\text{yd}$$

9. $\frac{500\text{in}}{\frac{12\text{in}}{\text{ft}}}$

$$12\frac{\text{in}}{\text{ft}}\overline{)500\text{in}}\ 41\text{ft}$$

48

20

12

8in

500in = 41ft 8in

$\frac{41\text{ft}}{\frac{3\text{ft}}{\text{yd}}}$

$$3\frac{\text{ft}}{\text{yd}}\overline{)41\text{ft}}\ 13\text{yd}$$

3

11

9

2ft

500in = 13yd 2ft 8in

10. 9 feet 3 inches = $9\text{ft}\ \frac{3\text{in}}{\frac{12\text{in}}{\text{ft}}} = 9\frac{1}{4}\text{ft}$ or 9.25ft

2 feet 9 inches = $2\text{ft}\ \frac{9\text{in}}{\frac{12\text{in}}{\text{ft}}} = 2\frac{3}{4}\text{ft}$ or 2.75ft

9.25ft x 2.75ft = 25.4375 sq ft

Alternate solution:

9 feet 3 inches = $9\text{ft} \times \frac{12\text{in}}{\text{ft}} + 3\text{in} = 108\text{in} + 3\text{in} = 111\text{in}$

2 feet 9 inches = $2\text{ft} \times \frac{12\text{in}}{\text{ft}} + 9\text{in} = 24\text{in} + 9\text{in} = 33\text{in}$

111in x 33in = 3663 sq in

$3663\ \text{sq in} \times \frac{1\ \text{sq ft}}{144\ \text{sq in}} = 25.4375\ \text{sq ft}$

LESSON 5

RATIOS AND PROPORTIONS

CREDIT HOURS ---------- 2

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Define a ratio.	5-1 (p.5-2)
2. Write the ratio expression in both written and verbal forms.	5-4 (p.5-8)
3. Identify the two parts of a ratio.	5-8 (p.5-16)
4. Apply the rules of ratios.	5-11 (p.5-22)
5. Set up and solve problems involving ratios.	5-15 (p.5-30)
6. Define a proportion.	5-41 (p.5-11)
7. Write the proportion expression in both written and verbal forms.	5-44 (p.5-17)
8. Identify the four parts of a proportion.	5-48 (p.5-25)
9. Apply the rules of proportions.	5-53 (p.5-35)
10. Set up and solve problems involving proportions.	5-59 (p.5-13)

SET 1. DEFINITION OF A RATIO

Frames 5-1 through 5-18 are at the top of even numbered pages.

FRAME 5-1

A RATIO is a RELATION or COMPARISON of one quantity to another quantity of the same kind. In mathematics the relation or a comparison of two similar quantities is called a _____.

Frames 5-19 through 5-36 are at the bottom of even numbered pages.

FRAME 5-19

Wrong! You failed to convert yards to inches before solving the unknown. Return to page 5-31, frame 5-68 and correct your solution and select another answer.

Frames 5-37 through 5-53 are at the top of odd numbered pages.

(a. $18.00; b. $27.00) (5-33)

FRAME 5-37

Very good! Let's check our understanding with another problem. Two fishermen agreed to divide a fish 3 feet 9 inches long in the ratio 4 to 5. How long is each section in inches?

a. Shorter _____ b. Longer _____

If your answer is:		Go to Frame:
a. $\frac{3}{5}$ feet $\frac{9}{5}$ inches	b. $\frac{3}{4}$ feet $\frac{9}{4}$ inches	5-18 (p. 5-36)
a. 2.08 feet	b. 1.67 feet	5-27 (p. 5-18)
a. 20 inches	b. 25 inches	5-39 (p. 5-7)

Frames 5-54 through 5-70 are at the bottom of odd numbered pages.

(2:5 = 4:8) (5-53)

FRAME 5-54

The rule of proportions that is used to solve for an unknown term in a proportion is _____

_____.

(ratio) (5-1)

FRAME 5-2

A knowledge of how "many" of a certain group may have little meaning in a discussion unless that quantity is related or compared with like or similar quantity by a ratio. Ratios are used to _____or_____two similar quantities.

FRAME 5-20

No. You have determined the ratio of the mathematics student to the other students in the school not taking mathematics. You obtained this answer by subtracting 96 (mathematics students) from (576) total number of students in the school and obtain a difference of 480 (other students). Your ratio was then 96:480 which reduces to 1:5. Return to page 5-30, frame 5-15, work the problem again, and select the correct answer.

FRAME 5-38

Wrong! You assumed that this was a direct proportion. Return to page 5-8, frame 5-22 and select another answer.

(the product of the extremes equals to the product of the means)

(5-51)

FRAME 5-55

There are two types of proportion, direct and inverse. Direct proportion is when an increase in one quantity causes a proportional increase in another quantity, or when a decrease in one quantity causes a proportional _____ in another quantity.

(relate; compare) (5-2)

FRAME 5-3

Write in your own words the definition of a ratio.

_____

_____

_____

[7/8] (5-25)

FRAME 5-21

$\frac{7}{8}$ (7:8) is the correct answer. Let's try and solve more difficult problems. Supposing that the sum of two numbers having a ratio of 1 to 3 is 32. What are the numbers? If you let X represent the smaller number and 3X the larger number, then X + 3X = 32. Solving for X, 4X = 32; X = 8. When X = 8, 3X = (3) (8) = 24. Checking, 8 + 24 = 32.

Go to page 5-28, frame 5-32.

(a. 20 inches; b. 25 inches) (5-37)

FRAME 5-39

Very good! Sometimes, more than two terms are used in ratio. The following problem contains three terms but it can be solved using the principles you already know. Mr. Smith left directions to divide his estate among three children in the ratios 2:3:4. If the estate amounted to $54,000, how much should each part be?

a. _____ b. _____ c. _____

(decrease) (5-55)

FRAME 5-56

Inverse proportion: When an increase in one quantity causes a proportional decrease in another quantity, or a decrease in one quantity causes a proportional _____ in another quantity.

SET 2. RATIO EXPRESSIONS

(A ratio is a relation or comparison of one quantity to another like or similar quantity) (5-3)

FRAME 5-4

A ratio is written in two different forms. One is in the form of a fraction with the two figures placed vertically and separated by a horizontal line across the middle. Example: A ratio of 2 to 3 is written $\frac{2}{3}$; hence, a ratio of 3 to 2 is written _____.

($162) (5-65)

FRAME 5-22

Right! Let's try another problem. If 15 carpenters can construct a building in 28 days, in how many days can 21 carpenters do the same job?

If your answer is:	Go to Frame:
12.5	5-30 (p. 5-24)
20.0	5-17 (p. 5-34)
39.2	5-38 (p. 5-5)

(a. 12,000; b. 18,000; c. 24,000 Solution in Frame below) (5-39)

FRAME 5-40

$2x + 3x + 4x = \$54,000$

$9x = \$54,000$

$x = 6,000$

$2x = 12,000 \qquad 3x = 18,000 \qquad 4x = 24,000$

This completes the frames on ratios. Now, proceed to frame 5-41 and the discussion of proportion.

(increase) (5-56)

FRAME 5-57

Six workmen make 1800 articles in one day. How many workmen would be needed to make 2700 such articles at the same rate? This problem is an example of a direct proportion because an increase in articles will require (a, an) _____ in workmen.

$\left[\frac{3}{2} \right]$ (5-4)

FRAME 5-5

A ratio may be written also with two figures placed horizontally and separated by a colon. Example: A ratio of 2 to 3 may be written 2:3; hence, a ratio of 3 to 2 is written_____.

FRAME 5-23

You're guessing or there is an error in your arithmetic. Go back to page 5-30, frame 5-33 and check your arithmetic and then select another answer.

SET 6. DEFINITION OF A PROPORTION

FRAME 5-41

Closely allied with the study of ratio is the subject of proportion. A proportion is nothing more than an equation in which the members are ratios. An equation in which the members are ratios is called _____.

(increase) (5-57)

FRAME 5-58

Six workmen completed a job in ten days. How many days will it take fifteen workmen to do the same job? This problem is an example of an inverse proportion because an increase in workmen will _____ the number of days.

(3:2) (5-5)

FRAME 5-6

A ratio of $\frac{1}{4}$ (1:4) is verbally expressed as <u>the ratio of one to four</u>. Then a ratio of $\frac{4}{1}$ (4:1) is verbally expressed as _____ _____ _____ _____ _____ _____.

FRAME 5-24

You're guessing! The numbers given in the problem could not give this answer. Return to page 5-31, frame 5-68 and work the problem and select another answer.

(proportion) (5-41)

FRAME 5-42

A proportion is an _____ in which the members are _____.

SET 10. SOLVING PROPORTION PROBLEMS

(decrease) (5-58)

FRAME 5-59

There are certain steps in setting up and solving problems involving direct proportions that simplify the procedure. The first step is to compare like things to like things in each of the ratio. In the problem "Six workmen make 1800 articles in one day. How many workmen would be needed to make 2700 such articles at the same rate?" To compare like things to like things, you would compare workmen are to _____ as articles are to _____.

(the ratio of four to one) (5-6)

FRAME 5-7

Write each of the following verbally expressed ratios as fractions and with colons.

a. The ratio of 5 lb to 15 lb is _____ and _____.

b. The ratio of $16 to $12 is _____ and _____.

(1:6) (5-15)

FRAME 5-25

1:6 ($\frac{1}{6}$) is the correct answer. Let's try another problem. In a class of 24 students, 3 students failed to pass the course. What is the ratio of students who passed the course to the whole class?

If your answer is:	Go to Frame:
$\frac{7}{8}$	5-21 (p. 5-6)
8:7	5-35 (p. 5-34)
1:8	5-31 (p. 5-26)

(equation; ratios) (5-42)

FRAME 5-43

Write the definition of a proportion.

_____

_____.

_____

(workmen; articles) (5-59)

FRAME 5-60

The second step is to place one set of related items as the antecedents of the two ratios and the other set as the consequent of the two ratios. Considering the problem mentioned in frame 5-59, the six workmen and 1800 articles would be the antecedents of the two ratios. The unknown number of workmen and the _____ articles
(number)
would be the consequents of the two ratios.

SET 3. PARTS OF A RATIO

[a. $\frac{5}{15}$ or 5:15; b. $\frac{\$16}{\$12}$ or \$16:\$12] (5-7)

FRAME 5-8

The two parts of a ratio are called the terms of a ratio, such as "first term" and "second term". In a ratio of $\frac{3}{4}$ or 3:4, the first term is 3, and 4 is the second term. In the ratio of $\frac{4}{3}$ or 4:3, 4 is the _____ _____ and 3 is the _____ _____.

FRAME 5-26

You haven't performed the arithmetic correctly. Return to page 5-34, frame 5-17 and check your work.

SET 7. PROPORTION EXPRESSIONS

(A proportion is an equation in which the members are ratios) (5-43)

FRAME 5-44

Proportions, like ratios, may be written in different forms. One is with the ratios terms in the form of fraction and with the ratios separated by an equal sign. Example: A proportion of 2 is to 4 as 4 is to 8 can be written $\frac{2}{4} = \frac{4}{8}$; hence, a proportion of 4 is to 2 as 8 is to 4 can be written _____.

(2700) (5-60)

FRAME 5-61

The third step is to solve for the unknown quantity in the direct proportion. The problem given on page 5-12, frame 5-59 would be set up and solved as follows:

	$\frac{6}{X} = \frac{1800}{2700}$
Multiply both sides by $\frac{X}{1}$	$\frac{6}{} = \frac{1800X}{2700}$
Multiply both sides by 2700	$(6)(2700) = 1800X$
Divide both sides by 1800	$\frac{(6)(2700)}{1800} = X$
Reduce	$X =$ _____

(First term; second term) (5-8)

FRAME 5-9

In a ratio, the first term is called the antecedent and the second term is called the consequent. Hence, when referring to a ratio of $\frac{4}{5}$ (4:5), 4 is the _____ and 5 is the _____.

FRAME 5-27

Wrong! You have reduced dimensions of the fish to a single unit which is good, but you have not read the problem carefully enough to know how to express the answer correctly. Return to page 5-3, frame 5-37 and reread the problem and select another answer.

$\left[\frac{4}{2} = \frac{8}{4}\right]$ (5-44)

FRAME 5-45

A proportion may also be written with the terms of each ratio written horizontally, separated by a colon, and with an equal sign separating the ratios. Example: A proportion of 2 is to 4 as 4 is to 8 can be written 2:4 = 4:8; hence, a proportion of 4 is to 2 as 8 is to 4 can be written _____.

(X = 9) (5-61)

FRAME 5-62

The first step in setting up an inverse proportion is the same as for a direct proportion. The first step is then to compare

_____

_____.

(antecedent; consequent) (5-9)

FRAME 5-10

The consequent of a ratio is the _____term and the antecedent of a ratio is the _____term.

FRAME 5-28

Wrong! You solved this proportion as if it were an inverse proportion. Go back to page 5-25, frame 5-65 and solve the proportion as a direct proportion.

(4:2 = 8:4) (5-45)

FRAME 5-46

A proportion $\frac{2}{4} = \frac{4}{8}$ or 2:4 = 4:8 is verbally expressed 2 is to 4 as 4 is to 8. Then, a proportion $\frac{3}{4} = \frac{6}{8}$ is verbally expressed:

_____.

(like things to like things in each ratio) (5-62)

FRAME 5-63

The second step in setting up inverse proportion, is to place one set of related items as the antecedents of the two ratios and the other set as the consequent of the two ratios and then to invert one of the ratios of the equation. It is important to recognize an inverse proportion. In the second step of the solution, it is necessary to _____ one of the ratios.

SET 4. APPLYING THE RULES OF RATIOS

(second; first) (5-10)

FRAME 5-11

Since ratio is similar to a fraction, all the rules that govern fractions may be used in working with ratios. Both terms of a ratio may be multiplied or divided by the same number without changing the value of the ratio. Examples:

$\frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$ $\frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$ $\frac{2}{4} = \frac{4}{8} = \frac{1}{2}$

Complete the following:

a. $\frac{4}{6} \times \frac{2}{2} =$ b. $\frac{4}{6} \div \frac{2}{2} =$

FRAME 5-29

Incorrect! You set up the ratio correctly but you failed to reduce it to its lowest terms. Return to page 5-30, frame 5-15, work the problem again, and select the correct answer.

(three is to four as six is to eight) (5-46)

FRAME 5-47

Write each of the following verbally expressed proportions in the fractional and horizontal form.

a. 15 is to 20 as 3 is to 4 is _____ and _____

b. 3 is to 8 as 9 is to 24 is _____ and _____

(invert) (5-63)

FRAME 5-64

The third step is to solve for the unknown quantity in the inverse proportion. Solve the following problem: Six workmen completed a job in ten days. How many days will it take fifteen workmen to do the same job?

6 workmen is to 15 workmen as 10 days is to X days

$$\text{workmen} \rightarrow \frac{6}{15} = \frac{10}{X} \leftarrow \text{days}$$

Inverse proportion: invert one ratio and solve for unknown

1. $\frac{6}{15} = \frac{X}{10}$

2. $15X = (6)(10)$

3. $X = \frac{(6)(10)}{15}$

4. $X =$ _____.

[a. $\frac{8}{12}$; b. $\frac{2}{3}$] (5-11)

FRAME 5-12

Like fractions, ratios should always be reduced to their lowest terms. A ratio is reduced to its lowest terms by dividing both terms by the same number until there is no whole number, other than one, that can be divided evenly into both. Example: $\frac{6}{9}$ may be reduced to $\frac{2}{3}$ by dividing both terms by 3. Write the following ratios in fractional form in their lowest term:

a. 4:16 b. $\frac{3}{18}$ c. 9:24

FRAME 5-30

Your guessing! The numbers given in this problem aren't related to this answer. Go back to page 5-8, frame 5-22 and solve the problem.

SET 8. PARTS OF A PROPORTION

$\left[\frac{15}{20} = \frac{3}{4}$ or $15:20 = 3:4$; $\frac{3}{8} = \frac{9}{24}$ or $3:8 = 9:24\right]$ (5-47)

FRAME 5-48

In a proportion $\frac{2}{3} = \frac{4}{6}$ or $2:3 = 4:6$, 2 is called the first term of this proportion, 3 the second term, 4 the third term, and 6 the fourth term. Then, in a proportion $\frac{2}{7} = \frac{4}{14}$ or $2:7 = 4:14$, the second and third terms are _____ and _____; the first and fourth terms are _____ and _____.

---

(4) (5-64)

FRAME 5-65

Set up and solve the following problem.

If 12 pieces of furniture cost $72, what will 27 pieces cost at the same rate?

If your answer is:	Go to Frame:
$32	5-28 (p. 5-20)
$162	5-22 (p. 5-8)
$200	5-34 (p. 5-32)

[a. $\frac{1}{4}$; b. $\frac{1}{6}$; c. $\frac{3}{8}$] (5-12)

FRAME 5-13

In finding the ratio of two numbers, both numbers must be first expressed in the same unit of measure. For instance, there can be no single ratio between 12 bolts and five men. A ratio should be yards to yards, quarts to quarts, and so on. Example: To find the ratio of 3 feet to 5 inches, the feet should first be converted to inches.

$$\frac{3\text{ft} \times 12\text{in/ft}}{5\text{in}} = \frac{36\text{in}}{5\text{in}} = \frac{7\frac{1}{5}}{1} \text{ or } \frac{7.2}{1} \text{ or } 7\tfrac{1}{5}:1 \text{ or } (7.2:1)$$

Find the ratio of the following, converting where necessary to the smaller unit of measure.

a. 2ft to 2yd b. 2in to 3yd c. 5mi to 3ft

FRAME 5-31

Wrong! You have set up the ratio of the students who failed the course to the whole class. Take a little more time reading the problem. Go back to page 5-14, frame 5-25 and find the correct answer.

(second = 7; third = 4; first = 2; fourth = 14) (5-48)

FRAME 5-49

In a proportion $\frac{2}{3} = \frac{4}{6}$ or 2:3 = 4:6, the first and fourth terms (2 and 6) are called the extremes, and the second and third terms (3 and 4) are called the means of this proportion. Then, in a proportion $\frac{5}{6} = \frac{10}{12}$ or 5:6 = 10:12, 6 and 10 are the _____; 5 and 12 are the _____.

(same) (5-16)

FRAME 5-66

In phrasing the verbal expression of a proportion for a map problem, you say: 1 is to 25,000 (for example, depends on map scale) as map distance is to ground distance. When setting a proportion for a map problem, the first term is always_____.

(a. 1:3; b. 1:54; c. 8800:1) (5-13)

FRAME 5-14

The value of a ratio is equal to the first term (antecedent) divided by the second term (consequent). The value of a ratio can be thought of as meaning how much larger or smaller one term is than another term with which it is being compared. Example: In a ratio $\frac{5}{8}$ or 5:8, 5 is divided by 8, to obtain the decimal value of .625. Determine the value of the following ratios:

a. $\frac{1}{4}$ b. $\frac{8}{3}$ c. 4:7

(You came from frame 5-21)

FRAME 5-32

Solve this problem: Sum of two numbers having a ratio of 4 to 7 is 99. What are the numbers?

a. Smaller no. _____ b. Larger no. _____

(means; extremes) (5-49)

FRAME 5-50

In a proportion, using the names for the terms of a ratio, the first ratio's _____ and second ratio's _____ are the extremes of the proportion. The first ratio's _____ and the second ratio's _____ are the means of the proportion.

(one or 1) (5-66)

FRAME 5-67

Let's try a problem. The scale of a map is 1:35,000. What is the ground distance if the distance on the map is 1.8 inches?

1 is to 35,000 as 1.8 inches (map distance) is to ground distance.

$$\frac{1}{35{,}000} = \frac{1.8 \text{ inches}}{X}$$

$$X = \frac{1.8 \times 35{,}000 \text{ inches}}{1}$$

Ground distances are normally expressed in feet, yards, meters, or miles.

$$X = \frac{1.8 \times 35{,}000 \text{ inches}}{\frac{12 \text{ inches}}{\text{foot}}}$$

X = _____ feet

SET 5. SOLVING RATIO PROBLEMS

(a. .250; b. 2.667; c. .571) (5-14)

FRAME 5-15

Let's put our knowledge of ratio to work in solving a practical problem. In a school last year, there were 576 students. 96 were in the mathematics course. What is the ratio of mathematics students to the whole school?

If your answer is:	Go to Frame:
$\frac{1}{5}$	5-20 (p. 5-4)
1:6	5-25 (p. 5-14)
$\frac{96}{576}$	5-29 (p. 5-22)

(4x + 7x = 99; 11x = 99; x = 9 therefore a. 4x = 36; b. 7x = 63) (5-32)

FRAME 5-33

Let's try another problem. Bob and Dick agreed to divide profits of $45.00 in the ratio of 2 to 3, with Dick securing the larger share. How much should each receive?

a. Bob _____ b. Dick _____

If your answer is:		Go to Frame:
a. $27.00	b. $18.00	5-36 (p. 5-36)
a. $18.00	b. $27.00	5-37 (p. 5-3)
a. $20.00	b. $25.00	5-23 (p. 5-10)

(antecedent; consequent; consequent; antecedent) (5-50)

FRAME 5-51

The first and fourth terms of a proportion are called the _____and the second and third terms of a proportion are called the _____.

(5,250 feet) (5-67)

FRAME 5-68

Let's try another problem. If the distance between two points on a map is 3.6 inches and the distance between the same two points on the ground is 3,520 yards, what is the scale of the map? Remember, the scale of the map is the first ratio of the proportion.

If your answer is:	Go to Frame:
1:9,800	5-19 (p. 5-2)
1:20,000	5-24 (p. 5-12)
1:35,200	5-69 (p. 5-33)

(4) (5-17)

FRAME 5-16

Right! Another type of problem where proportions are useful is in the conversion of map distance to ground distance. Each map has a scale, for example 1:25,000. This is the first ratio in the proportion. The second is the ratio of map distance to ground distance. The map distance and ground distance must be expressed in the _____units.

Answer is above frame 5-66 (p. 5-27)

FRAME 5-34

Your guessing! The numbers given in this problem aren't related to this answer. Go back to page 5-25, frame 5-65 and solve the problem.

(extremes; means) (5-41)

FRAME 5-52

If the means of a proportion are the same term, this term is called the mean proportional between the other two terms. In the proportion 2:8 = 8:32, 8 is the mean proportional between 2 and 32. In the proportion 1:7 = 7:49, 7 is the _____ _____.

FRAME 5-69

Very good!

$$\frac{1}{X} = \frac{3.6}{3520 \times 36}$$

$$3.6X = 126720$$

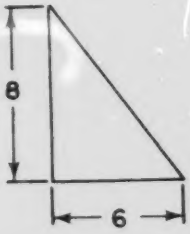
$$X = 35{,}200$$

Map scale 1:35,200

(20.0) (5-22)

FRAME 5-17

Very good! One type of problem where proportions are useful is in the solution of similar geometric figures. For example, the sides of similar triangles are in proportion. Given that A and B are similar triangles, what is the value of X? The proportion is stated 6 is to 8 as 3 is to X.



If your answer is:	Go to Frame:
5	5-26 (p. 5-16)
4	5-16 (p. 5-32)

FRAME 5-35

The only way possible to get this answer you have is to set up the ratio of students in the class to the students that passed. Read the problem carefully. Return to page 5-14, frame 5-25 and work the problem correctly. Then select the correct answer and go to the indicated frame.

SET 9. RULES OF PROPORTIONS

(mean proportional) (5-52)

FRAME 5-53

In a proportion the product of the extremes is equal to the product of the means. Example: In proportion $\frac{3}{5} = \frac{6}{10}$ or 3:5 = 6:10, product of extremes (3 and 10) equals 30 and product of means (5 and 6) equals 30. Which one(s) of the following is(are) not a proportion?

a. 2:5 = 4:8 b. $\frac{3}{5} = \frac{9}{15}$ c. $\frac{2}{3} = \frac{6}{9}$

Turn back to the bottom of page 5-3.

FRAME 5-70

You have completed the programmed lesson on ratio and proportion. They are a very useful mathematical tool for comparing similar quantities. Turn to the SELF-TEST on page 5-37 and check your understanding of ratios and proportions.

FRAME 5-18

You have made two errors in your solution. It is better to express the terms of the ratio in a single unit (either feet or inches in the problem). Secondly, you have not performed the operation with the ratio correctly. Review page 5-6, frame 5-21 and then return to page 5-3 frame 5-37 and select another answer.

FRAME 5-36

You didn't read the problem carefully. You inverted the ratio. Return to page 5-30, frame 5-33 and determine which boy was to receive the larger share, then select another answer.

SELF-TEST ON RATIOS AND PROPOkTIONS

1. Write the definition of a ratio.

_____

_____

2. Write each of the following verbally expressed ratios in both standard written forms.

   a. the ratio of 2 to 14 _____ or _____

   b. the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ _____ or _____

3. The first term of a ratio is called the _____.

4. The ratio of 8oz to 21b is _____.

5. The value of the ratio $\frac{2}{5}$ or 2:5 is _____.

6. Divide 80 trucks between two sergeants in the ratio of 5:3.

   First sergeant _____ and second sergeant_____.

7. Write the definition of a proportion.

_____

_____

8. Write each of the following verbally expressed proportions in both of the standard written forms.

   a. 3 is to 5 as 9 is to 15 _____ and _____

   b. $\frac{1}{4}$ is to $\frac{1}{2}$ as 1 is to 2 _____ and _____

9. In the proportion $\frac{2}{5} = \frac{8}{20}$, 2 and 20 are the _____ and 5 and 8 are the _____.

10. Write the rule of proportions that is used to solve for an unknown term in a proportion.

_____

_____

11. The two types of proportions are _____ and _____.

12. A man 6 feet tall casts a shadow 8 feet long. If he is standing by a flag pole, and its shadow is 100 feet, how tall is the pole? _____

13. Your map has a scale of 1:50,000. What is the distance on the map in inches if the distance on the ground is 3750 feet?

_____

ANSWERS TO SELF-TEST LESSON 5

1. Ratio is a relation or comparison of one quantity to another quantity of the same kind.

2. a. $\frac{1}{7}$ or 1:7 b. $\frac{3}{2}$ or 3:2

3. antecedent

4. $\frac{8oz}{2lb} = \frac{8oz}{2lb \times \frac{16oz}{1lb}} = \frac{8oz}{32oz} = \frac{1}{4}$

5. $\frac{2}{5} = .4$

6. $5X + 3X = 80$

   $8X = 80$

   $X = 10$

   First sergeant $5X = 5(10) = 50$

   Second sergeant $3X = 3(10) = 30$

7. A proportion is an equation in which the members are ratios.

8. a. $\frac{3}{5} = \frac{9}{15}$ or 3:5 = 9:15

   b. $\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$ or $\frac{1}{4} : \frac{1}{2} = 1:2$

9. extremes; means

10. In a proportion the product of the extremes is equal to the product of the means.

11. direct; inverse

12. 6 feet is to 8 feet as X feet is to 100 feet

$$\frac{6}{8} = \frac{X}{100}$$

$$X = \frac{6 \times 100}{8}$$

$$X = 75 \text{ feet}$$

13. 1 is to 50,000 as X is to 3750 feet

$$\frac{1}{50{,}000} = \frac{X}{3750\text{ft} \times 12\frac{\text{in}}{\text{ft}}}$$

$$X = \frac{3750 \times 12\text{in}}{50{,}000}$$

$$X = .9 \text{ inch}$$

LESSON 6

PERCENTAGE

CREDIT HOURS ---------- 1

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Write the definition of percent.	6-1 (p.6-2)
2. Change decimals to percents.	6-6 (p.6-12)
3. Change percents to decimals.	6-12 (p.6-24)
4. Change percents to fractions.	6-16 (p.6-6)
5. Change fractions to percent.	6-19 (p.6-12)
6. Substitute numbers taken from given problems into the "percentage problem formula". The formula will be given.	6-51 (p.6-25)
7. Solve problems by finding percentages.	6-27 (p.6-3)
8. Solve problems by finding what percent one number is of another.	6-30 (p.6-9)
9. Solve problems by finding a number when the percent of a number is known.	6-32 (p.6-13)
10. Solve problems where the percents are greater than 100%.	6-34 (p.6-17)

SET 1. DEFINITION OF PERCENT

Frame 6-1 through Frame 6-13 are on the top of even numbered pages.

FRAME 6-1

Percent (%) is defined as parts per hundred. Therefore, in the fraction $\frac{6}{100}$, which is the same as the decimal .06, the 6 indicates we are concerned with six parts per _____.

(percent sign) (6-13)

FRAME 6-14

The second step in changing a percent to a decimal is to move the decimal point _____places to the _____.

SET 7. FINDING PERCENTAGES

$\left[\frac{x}{20} = \frac{5\%}{100\%} \right]$ (6-26)

FRAME 6-27

You are now going to learn how to solve percentage problems. For example, 20 students are in the class. 20% are failing. How many students are failing? Substitute into the formula, change percents to decimals, and solve for the unknown. (Do not reduce the fraction before solving for the unknown.)

Step #1 Read the problem carefully

#2 $\frac{\text{small}}{\text{large}} = \frac{\%}{100\%}$ (Set up formula)

#3 $\frac{\text{X (unknown)}}{\text{20 students}} = \frac{20\%}{100\%}$ (Substitute into the formula)

#4 $\frac{X}{20} = \frac{.20}{1.00}$ (Change percents to decimals)

#5 $X = \frac{20 \times .20}{1.00}$ (Solve for the unknown)

#6 X = _____ students failing.
(Your answer)

$\left[\frac{63}{720} = \frac{X}{100\%} \right]$ (6-46)

FRAME 6-40

You have set the problem up in the correct form for solving percents. Try one more for a double check. "John purchased 24 oranges and later found 6 were spoiled. What percent were spoiled?" Select the correct formula.

If your answer is:	Go to Frame:
$\frac{6}{24} = \frac{X}{100\%}$	6-49 (p. 6-21)
$\frac{6}{X} = \frac{100\%}{24}$	6-44 (p. 6-11)

(hundred) (6-1)

FRAME 6-2

33%, written as a fraction $\frac{33}{100}$ or the decimal .33, indicates we are concerned with _____parts per _____.

(two; left) (6-14)

FRAME 6-15

Change the following percents to decimals:

a. 34% equals _____

b. $\frac{1}{4}$% equals _____

c. 24.5% equals _____

d. .5% equals _____

(4) (6-27)

FRAME 6-28

Solve the following problems. During the month of April (30 days), it rained 23% of the time. How many days did it rain? You are to solve for the number of days out of 30 days that 23% represents.

_____

(Place answer here)

FRAME 6-41 (You came from frame 6-37)

Let's try another problem. If there are 90 people in a town and 10% of them are farmers, what number should go on the bottom left to complete the formula?

$$\frac{\text{small}}{} = \frac{10\%}{100\%}$$

If your answer is:	Go to Frame:
9	6-47 (p. 6-17)
10%	6-38 (p. 6-25)
90	6-43 (p. 6-9)

(33; hundred) (6-2)

FRAME 6-3

5% of a number means $\frac{5}{100}$ of it; 15% of a number means $\frac{15}{100}$ of it. Therefore, 23% of a number means _____ (fraction) of it and indicates we are concerned with _____ parts per hundred.

SET 4. CHANGE PERCENTS TO FRACTIONS

(a. .34; b. .0025; c. .245; d. .005 (Read the question carefully. $\frac{1}{4}\%$ is less than 1%. Therefore, $\frac{1}{4}\%$ equals the decimal .0025 and not .25)) (6-15)

FRAME 6-16

To change a percent to a common fraction, first change the percent to a decimal. For example, 45% would be changed to the decimal .45. The second step after changing the percent to a decimal is to change the decimal to a fraction and reduce to its lowest terms. For example, $80\% = .80 = \frac{80}{100}$ or $\frac{4}{5}$. In changing a percent to a fraction, first change the percent to a _____ and then to a _____ and reduce to its lowest terms.

$$\left[\frac{X}{30} = \frac{23\%}{100\%} ; \frac{X}{30} = \frac{.23}{1.00} ; X = \frac{.23 \times 30}{1.00} ; X = 6.9\right] \quad (6\text{-}28)$$

FRAME 6-29

During the year (365 days), it was cloudy 48% of the time. How many days was it cloudy?

_____

(Place answer here)

FRAME 6-42

25 is not correct. 25 is the total number of soldiers in the office. Therefore, the number of soldiers that are sergeant must be a smaller number. Return to page 6-25 frame 6-51, and select another answer.

$\left[\frac{23}{100}\ ;\ 23\right]$ (6-3)

FRAME 6-4

Percent is defined as _____ per _____.

(decimal; fraction) (6-16)

FRAME 6-17

60% is equal to the decimal _____ and the fraction _____.

SET 8. FINDING WHAT PERCENT ONE NUMBER IS OF ANOTHER

$\left[\frac{X}{365} = \frac{48\%}{100\%} \text{ or } X = 175.2\right]$ (6-29)

FRAME 6-30

Solve the following problems by finding what percent one number is of another. It rained 12 days during the months of September and November (60 days). What percent of the total days did it rain?

(Place answer here)

---

(90) (6-41)

FRAME 6-43

90 is correct. 90 is the total number of people in town. The number of farmers would be a smaller number, since there are only 10% farmers. Proceed to page 6-15, frame 6-46.

(Parts; hundred) (6-4)

FRAME 6-5

In your own words, write the definition of percent.

_____.

_____

[.60 or $\frac{60}{100}$ or $\frac{3}{5}$] (6-17)

FRAME 6-18

Change the following percents, first to decimals and then to fractions. Reduce to lowest terms.

a. 40% = _____ = _____

b. 22.5% = _____ = _____

c. 20% = _____ = _____

d. 12.5% = _____ = _____

$\left[\frac{12}{60} = \frac{X}{100\%} \text{ or } X = 20\%\right]$ (6-30)

FRAME 6-31

Of the 1400 personnel attached to the Army Post, 65% own automobiles. How many personnel own automobiles?

_____
(Place answer here)

FRAME 6-44

$\frac{6}{X} = \frac{100\%}{24}$ is incorrect. All percents are placed on one side of the equal sign and all other information on the opposite side. Return to page 6-3, frame 6-40 and select another answer.

SET 2. CHANGE DECIMALS TO PERCENTS

(parts per hundred) (6-5)

FRAME 6-6

To change a decimal to a percent, the first step is to move the decimal point two places to the right. Therefore, the decimal .28 becomes 28. . To change a decimal to a percent, the first step is to move the decimal point two places to the _____ .

SET 5. CHANGE FRACTIONS TO PERCENTS

[a. .40 or $\frac{2}{5}$; b. .225 or $\frac{9}{40}$; c. .20 or $\frac{1}{5}$; d. .125 or $\frac{1}{8}$] (6-18)

FRAME 6-19

To change a fraction to a percent, first change the fraction to a decimal and then the decimal to a percent. For example, $\frac{3}{8}$ = .375 = 37.5%. To change the fraction $\frac{2}{3}$ to a percent, you must first change it to a _____.

SET 9. FINDING A NUMBER WHEN THE PERCENT OF A NUMBER IS KNOWN

$\left[\frac{X}{1400} = \frac{65\%}{100\%} \text{ or } X = 910\right]$ (6-31)

FRAME 6-32

Solve the following problems by finding a number when the percent of a number is known:

The Smith family spends $52 weekly on food, shelter, and clothing. That is 65% of their weekly income. What is their weekly income?

FRAME 6-45

$\frac{63}{720} = \frac{100\%}{X}$ is incorrect. You are looking for the smaller percentage. 100% should be on the bottom of the formula. Return to page 6-15, frame 6-46, and select another answer.

(right) (6-6)

FRAME 6-7

After moving the decimal point two places to the right, add the percent sign (%). Therefore, the decimal .28 becomes 28%. To change a decimal to a percent, move the decimal point _____ places to the _____ and add the _____ sign.

(decimal) (6-19)

FRAME 6-20

After changing the fraction to a decimal, change the decimal to a percent. $\frac{3}{4}$ is changed to the decimal .75 and then changed to a _____.

$\left[\frac{\$52}{\$X} = \frac{65\%}{100\%} \text{ or } X = \$80\right]$ (6-32)

FRAME 6-33

Bill said that the $10.50 he intended to spend for a football was just 35% of what he had saved. How much did he save?

FRAME 6-46

Determine the correct formula for the solution of the following problem. "X" indicates the unknown number (number to be found). Out of 720 weather observations taken last month, there were 63 errors. What was the percent of error? You are looking for a percent that represents 63 errors.

If your answer is:	Go to Frame:
$\frac{63}{X} = \frac{100\%}{720}$	6-39 (p. 6-27)
$\frac{63}{720} = \frac{100\%}{X}$	6-45 (p. 6-13)
$\frac{63}{720} = \frac{X}{100\%}$	6-40 (p. 6-3)
$\frac{X}{720} = \frac{63}{100\%}$	6-50 (p. 6-23)

(two; right; %) (6-7)

FRAME 6-8

A decimal point is not shown with whole percents. For example, 63% does not have a decimal point shown. However, a decimal point is placed mentally to the right of the number three (63.%). In the percent below, place an "X" where you would mentally place a decimal point.

47 %

(percent) (6-20)

FRAME 6-21

To change a fraction to a percent, first change the fraction to a _____ and then to a _____.

SET 10. PERCENTS GREATER THEN 100%

$\left[\frac{\$10.50}{X} = \frac{35\%}{100\%} \text{ or } X = \$30\right]$ (6-33)

FRAME 6-34

At times, you will be using percents greater than 100%. For example, current enrollment is 250% of last year's; last year's enrollment was 160. In this case, the larger percent (250%) goes on the bottom and 100% goes on top. For example, $\frac{160}{X} = \frac{100\%}{250\%}$. Solve the following problems where percents are greater than 100%:

Last year the donations amounted to $154. This year's donations were 120% of last year's. What was the amount donated this year?

FRAME 6-47

9 is incorrect. You have solved the problem correctly for the number of people in the town that are farmers, but that is not the question to be answered. Return to page 6-5, frame 6-41 and read the frame again and select another answer. Remember in percentage, the larger number (total people) goes on the bottom or in the denominator.

(47 x %) (6-8)

FRAME 6-9

Fractional percents do have decimal points. For example, $56\frac{1}{2}\%$ can be written as 56.5%. The fractional percent $23\frac{1}{4}\%$ can be written as _____.

(decimal; percent) (6-21)

FRAME 6-22

$\frac{4}{5}$ is first changed to the decimal .80 and then to the percent _____.

$\left[\frac{\$154}{X} = \frac{100}{120} \text{ or } X = \$184.80\right]$ (6-34)

FRAME 6-35

Find 122% of 65.

FRAME 6-48

100% is not correct. You are looking for the smallest number. 100% would be the total percent of soldiers in the office or the largest number percentage-wise. Return to page 6-25, frame 6-51 and select another answer.

(23.25%) (6-9)

FRAME 6-10

The decimal .83 is changed to a percent by moving the decimal point _____ places to the _____ and adding the percent sign. The result is _____.

[80% ($\frac{4}{5}$) is changed to a decimal by dividing the 4 by 5 and then to a percent by moving the decimal point two places to the right and adding the percent sign %.] (6-22)

FRAME 6-23

Change the following fractions to percents. Be careful with your division.

a. $\frac{3}{4}$ = _____

b. $\frac{3}{8}$ = _____

c. $\frac{7}{8}$ = _____

d. $\frac{3}{9}$ = _____

NOTE: Go to page 6-25, Frame 6-51 for the answers.

$\left[\frac{65}{X} = \frac{100\%}{122\%} \text{ or } X = 79.3\right]$ (6-35)

FRAME 6-36

You have completed a programmed course in percentage. A SELF-TEST starts on page 6-28.

$\left[\frac{6}{24} = \frac{X}{100\%}\right]$ (6-40)

FRAME 6-49

You selected the correct formula. Now for the final test. Go to page 6-22, frame 6-24, and continue.

(two; right; 83%) (6-10)

FRAME 6-11

Change the following decimals to percents:

a. .15 equals _____%

b. .23 equals _____%

c. .75 equals _____%

d. .31 equals _____%

e. .005 equals _____%

FRAME 6-24

A company had 100 employees and 15 were women. Write the formula for determining the percent of women employees, using the given information. Proceed to the next frame to check your answer.

_____
(Your answer)

(5) (6-51)

FRAME 6-37

5 is correct. 5 is the smallest number and it represents the part of the total number of soldiers that are sergeants. Proceed to page 6-5, frame 6-41.

FRAME 6-50

$\frac{X}{720} = \frac{63}{100\%}$ is incorrect. Remember that all percents are on one side of the equal sign. The unknown (X) is a percent and should be on the other side. Return to page 6-15, frame 6-46, and select another answer.

SET 3. CHANGE PERCENTS TO DECIMALS

[a. 15%; b. 23%; c. 75%; d. 31%; e. $\frac{1}{2}$% or .5%] (6-11)

FRAME 6-12

A percent is changed to a decimal by dropping the percent sign and moving the decimal point two places to the left. For example, 12.5% becomes 12.5 after dropping the percent sign. When the decimal point is moved two places to the left, 12.5 then becomes .125. Therefore, the percent 12.5% equals the decimal .125. In changing a percent to a decimal, move the decimal point two places to the

_____.

$\left[\frac{15}{100} = \frac{X}{100\%}\right]$ (6-24)

FRAME 6-25

It rained 10 days out of 30. Write the formula for determining percent of rain days, using the given information.

_____

FRAME 6-38

10% is wrong. 10% is the smaller of the two percents. However, we are looking for the number of people in the town. Return to page 6-5, frame 6-41, and select another answer.

SET 6. PERCENTAGE PROBLEM FORMULA

(a. 75%; b. 37.5%; c. 87.5%; d. 33.3%) (6-23)

FRAME 6-51

All percentage problems can be solved by substituting given information into the formula $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$.

For the problem, "There are 25 soldiers in the office. Of the 25, there are 5 sergeants. What percent of the soldiers are sergeants?", the formula would look like this: $\frac{(\text{small})}{(\text{large})} = \frac{X(\text{unknown})\%}{100\%}$

Substituting from the information in the problem, what number should you place on the top left in the formula?

If your answer is:	Go to Frame:
5	6-37 (p. 6-23)
25	6-42 (p. 6-7)
100%	6-48 (p. 6-19)

(left) (6-12)

FRAME 6-13

The first step in changing 46% to a decimal is to drop the

_____ _____.

Turn back to the bottom of page 6-2.

$\left[\frac{10}{30} = \frac{X}{100\%}\right]$ (6-25)

FRAME 6-26

20 men work in the weather office. 5% can go on leave. Write the formula for determining the number of men that can go on leave. Remember, percentage numbers all go on one side of the formula and all other information on the other side. For example:

$$\frac{\text{small}}{\text{large}} = \frac{\text{small \%}}{\text{large \%}}$$

Turn back to the top of page 6-3.

FRAME 6-39

$\frac{63}{X} = \frac{100\%}{720}$ is incorrect. Remember to keep all the percents on one side of the equal sign and all other information on the other side. Also, the smaller numbers should be on top and the larger numbers on the bottom. Return to page 6-15, frame 6-46, and select another answer.

SELF -TEST

1. Write the definition of percent.

---

2. Change each of the following decimals to a percent:

   a. .12 =

   b. .23 =

   c. .005 =

3. Change each of the following percents to a decimal:

   a. 28% =

   b. $63\frac{1}{2}\%$ =

   c. $\frac{3}{4}\%$ =

4. Change each of the following percents to a fraction and reduce to lowest terms:

   a. 73% =

   b. 20% =

   c. 43% =

5. Change each of the following fractions to a percent:

   a. $\frac{3}{4}$ =

   b. $\frac{2}{3}$ =

   c. $\frac{13}{16}$ =

6. Using the "percentage problem formula", $\frac{\text{small number}}{\text{large number}} = \frac{\%}{100\%}$, substitute numbers into the formula from the following problems. Do not try to solve the problems. Set up the formula only.

   a. The trainee fired 50 shots at the target, missing only 3%.

   b. During the next firing, the trainee missed 12 out of 75 shots.

   c. 9 privates work in an office. The privates make up 12% of the office staff.

   a. _____ b. _____ c. _____

7. Solve the following problems by finding the percentage. Use the formula.

   a. Last month the Weather Office took 720 weather observations and made 6% error. How many errors were made?

   b. Out of 40 personnel in the Weather Office, 80% are qualified weather observers. How many personnel are qualified weather observers?

8. Solve the following problems by finding what percent one number is of another:

   a. The duty section made 14 errors out of 60 observations. What was their percent of error?

   b. The weather office has 4 out of 32 personnel on sick call. What percent of the total personnel are on sick call?

9. Solve the following problems by finding a number when the percent of a number is known:

   a. 10% of the weather office personnel are allowed on leave. If there are 4 men on leave, how many men are attached to the weather office?

   b. $\frac{3}{4}$ % of the observations are in error. There are 12 errors. How many observations were taken?

10. Solve the following problems. The percents are greater than 100%.

   a. Last year the enrollment at the school was 540 students. The enrollment this year is 125% of last year's. What was the enrollment this year?

   b. Find 275% of 44.

ANSWERS TO SELF-TEST LESSON 6

1. Parts per hundred.

2. To change a decimal to a percent, move the decimal point two places to the right and add the percent sign.

   a. 12% b. 23% c..5%

3. To change a percent to a decimal, drop the percent sign and move the decimal point two places to the left.

   a. .28 b. .635 c. .0075

4. To change a percent to a fraction, change the percent to a decimal and the decimal to a fraction and reduce to lowest terms.

   a. 73% .73 $\frac{73}{100}$

   b. 20% .20 $\frac{20}{100} = \frac{1}{5}$

   c. 43% .43 $\frac{43}{100}$

5. To change a fraction to a percent, change the fraction to a decimal and the decimal to a percent.

   a. $\frac{3}{4} = .75$ 75%

   b. $\frac{2}{3} = .66\frac{2}{3}$ $66\frac{2}{3}\%$

   c. $\frac{13}{16} = .8125$ 81.25%

6. a. $\frac{X}{50} = \frac{3\%}{100\%}$ b. $\frac{12}{75} = \frac{X}{100\%}$ c. $\frac{9}{X} = \frac{12\%}{100\%}$

7. a. $720 \times .06 = 43.2$

b. $40 \times .80 = 32$

8. a. $\frac{14}{60} = \frac{X}{100\%}$

$X = \frac{14 \times 100}{60}$

$X = 23.3\%$

b. $\frac{4}{32} = \frac{X}{100\%}$

$X = \frac{4 \times 100}{32}$

$X = 12.5\%$

9. a. $\frac{4}{X} = \frac{10\%}{100\%}$

$X = \frac{4 \times 100\%}{10\%}$

$X = 40$

b. $\frac{12}{X} = \frac{3/4\%}{100\%}$

$X = \frac{12 \times 100\%}{3/4\%}$

$X = 1600$

10. a. $\frac{540}{X} = \frac{100\%}{125\%}$

$X = \frac{540 \times 125\%}{100\%}$

$X = 675$

b. $\frac{44}{X} = \frac{100\%}{275\%}$

$X = \frac{44 \times 275\%}{100\%}$

$X = 121$

LESSON 7

POWERS AND ROOTS

CREDIT HOURS ---------- 2

TEXT ASSIGNMENT ---------- Programmed Text

MATERIALS REQUIRED ---------- Pencil

LESSON OBJECTIVES

The student will:	Begin at FRAME
1. Write the definition of a power of a number.	7-1 (p.7-2)
2. Write the names of the elements used to indicate the arithmetic operation of raising a number to a power.	7-6 (p.7-12)
3. Write the definition of the square of a number.	7-8 (p.7-16)
4. Select the numbers that are perfect squares from a given list of numbers.	7-14 (p.7-28)
5. Write the square of each number in a given list.	7-17 (p.7-34)
6. Write the names of higher powers of numbers.	7-18 (p.7-2)
7. Write the definition of the square root of a number.	7-20 (p.7-6)
8. Solve problems by finding the square root from a given list of whole numbers, fractions, and decimals.	7-24 (p.7-14)
9. Check a given square root problem for accuracy.	7-58 (p.7-17)
10. Solve problems by finding the root of perfect powers.	7-62 (p.7-25)

SET 1. DEFINITION OF A POWER OF A NUMBER

FRAMES 7-1 through 7-17 are on the top of even numbered pages.

FRAME 7-1

When several numbers are multiplied together, as $2 \times 3 \times 4 = 24$, the numbers 2, 3, and 4 are factors and 24 is the product. In $1 \times 2 \times 5 = 10$, the numbers 1, 2, and 5 are _____ and 10 is the _____.

SET 6. HIGHER POWERS OF NUMBERS

Frames 7-18 through 7-34 are on the bottom of even numbered pages.

[a. 100; b. $\frac{4}{9}$; c. 64; d. 1.96; e. $\frac{9}{16}$; f. 25; g. .0144; h. .16]

(7-17)

FRAME 7-18

The square is the second power of a number. The third power of a number is called the cube of the number. 5^3 is called the _____ of 5.

Frames 7-35 through 7-50 are on the top of odd numbered pages.

Answer to frame 7-34

Sum = 43

Product = 129

The problem should resemble this:

$$\begin{array}{r} 2\ \ 3. \\ \sqrt{5\ \ 29.} \\ \underline{4\ \ \ \ } \\ 1\ \ 29 \end{array}$$

20x2 = 40
+ 3
43
x 3
129

FRAME 7-35

When you obtain the product, it is written beneath the dividend and subtracted from the dividend.

In the problem below, write the product (129) beneath the dividend and subtract.

$$\begin{array}{r} 2\ \ 3. \\ \sqrt{5\ \ 29.} \\ \underline{4\ \ \ \ } \\ 1\ \ 29 \end{array}$$ 1st dividend

20 x 2 = 40
+ 3
43
x 3
129

Frames 7-51 through 7-66 are on the bottom of odd numbered pages.

Answer to frame 7-50

You should have this:

$$\begin{array}{r} 2\ \ 5.\ 2\ \ 1 \\ \sqrt{6\ 36.00\ 00} \\ \underline{4\ \ \ \ \ \ \ \ } \\ 2\ 36 \\ \underline{2\ 25\ \ \ \ } \\ 11\ 00 \\ \underline{10\ 04\ } \\ 96\ 00 \end{array}$$

20x252 = 5040

FRAME 7-51

Add the estimate to the 3rd trial divisor and multiply that sum by the estimate.

$$\begin{array}{r} 2\ \ 5.\ 2\ \ 1 \\ \sqrt{6\ 36.00\ 00} \\ \underline{4\ \ \ \ \ \ \ \ } \\ 2\ 36 \\ \underline{2\ 25\ \ \ \ } \\ 11\ 00 \\ \underline{10\ 04\ } \\ 96\ 00 \end{array}$$

3rd trial divisor

20 x 252 = 5040

(factors; product) (7-1)

FRAME 7-2

If all the factors are alike, as 2 x 2 x 2 x 2, then the product, 16, is known as a POWER of 2 and 2 is the BASE of that power. When a base is multiplied by itself a certain number of times, the product is known as a _____ of that base.

(cube) (7-18)

FRAME 7-19

The higher powers of a number have no special names. For example, 3^4, is called the fourth power of 3.

6^7 is called the _____power of 6.

Answer to frame 7-35

You should have this:

```
          2  3.
       ---------
    √ (5) (29.)
          4
       ---------
20x2 = 40   1 29
    + 3     1 29
    ----    ----
      43    0 00
    x 3
    ---
     129
```

FRAME 7-36

In the problem above, the square root extracts evenly. Now, let's work one where the square root does not extract evenly and there is a remainder other than zero. Follow the instructions closely.

Arrange the number below for extraction of its square root.

636

Answer to frame 7-51

You should have this:

```
          2  5.  2  1
       ------------
    √ (6) (36.00) (00)
          4
       ---------
       2  36
       2  25
       -------
         11  00
         10  04
         -------
             96  00
    ---------------
5040
+  1
----
5041
x  1
----
5041
```

FRAME 7-52

Place the product obtained in the preceding frame beneath the 3rd dividend and subtract.

```
          2  5.  2  1
       ------------
    √ (6) (36.00) (00)
          4
       ---------
       2  36
       2  25
       -------
         11  00
         10  04
         -------
             96  00
    ---------------
20 x 252 = 5040
        +    1
        ------
          5041
        x    1
        ------
          5041
```

(power) (7-2)

FRAME 7-3

A power is a product obtained by using a base a certain number of times as a factor. If all the factors in a multiplication operation are alike, then the product is known as a power and the common factor (number multiplied by itself) is known as the _____ of that power.

SET 7. DEFINITION OF THE SQUARE ROOT OF A NUMBER

(seventh) (7-20)

FRAME 7-20

The inverse (opposite) of raising a number to a power is called extracting the root. When a number is equal to the product of two or more equal factors, the equal factor is known as the ROOT of the number. For example, the two equal factors of 16 are 4 and 4 (4 x 4 = 16). Therefore, 4 is a root of 16. In a like manner 2, 2, and 2, are the three equal factors of 8.

Therefore for 8, 2 is a _____.

Answer to frame 7-36

$\sqrt{6\ 36.}$

FRAME 7-37

In the problem below, find the largest perfect square which is equal to or less than the first digit in the radicand.

Write the perfect square below t..e first digit and its square root directly above the first digit.

$\sqrt{6\ 36.}$

Answer to frame 7-52

You should have this:

$$\begin{array}{r} 2\ \ 5.\ 2\ \ 1 \\ \sqrt{6\ 36.00\ 00} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \\ 10\ 04 \\ \hline 96\ 00 \\ 50\ 41 \\ \hline 45\ 59 \end{array}$$

5040
+ 1
5041
x 1
5041

Remainder

FRAME 7-53

You have been shown step by step how to get necessary decimal places in extracting a square root. Now, to the problem below, add necessary zeros to the radicand and perform the steps required to find the third decimal place in the square root.

$$\begin{array}{r} 2\ \ 5.\ 2\ \ 1 \\ \sqrt{6\ 36.00\ 00} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \\ 10\ 04 \\ \hline 96\ 00 \\ 50\ 41 \\ \hline 45\ 59 \end{array}$$

NOTE: Answer to frame 7-53 is in panel 7-1, page 7-35.

(base) (7-3)

FRAME 7-4

The base is not limited to just whole numbers. Fractions, decimals, and even letters can be used as the base. The product obtained by using a base a certain number of times as a factor is known as the _____of that _____.

(root) (7-20)

FRAME 7-21

When a number has two equal factors, either one of the factors is called the SQUARE ROOT of the number. For example, the square root of 16 is 4. Four (4) is one of the two equal factors that, when multiplied together, give you 16.

The square root of 25 is _____, which is one of two equal factors you can _____together to give you 25.

Answer to frame 7-37

You should have this:

$$\begin{array}{r} 2\ . \\ \sqrt{6\ 36.} \\ 4 \end{array}$$

FRAME 7-38

Continue the problem below by subtracting the perfect square from the first digit; then bring the next pair of digits in the radicand down beside the remainder.

$$\begin{array}{r} 2\ . \\ \sqrt{6\ 36.} \\ \underline{4} \end{array}$$

1st dividend

Answer to frame 7-53

Correct response to frame 7-53 is in panel 7-1, page 7-35.

FRAME 7-54

As you can see by the preceding problem, when the square root does not extract evenly, you can carry it out to the number of decimal places necessary for the required accuracy.

Find the square root of the number below. Carry your answer 3 places to the right of the decimal.

542.071

(power; base) (7-4)

FRAME 7-5

Write in your own words the definition of power.

_____

_____

_____

(5; multiply) (7-21)

FRAME 7-22.

One of the two equal factors, which, when multiplied together, give us a number, is the _____ _____ of that number.

Answer to frame 7-38

You should have this:

$$\begin{array}{r} 2 \\ \sqrt{\overline{6}\,\overline{36.}} \\ \underline{4} \\ 2 \quad 36 \end{array}$$

FRAME 7-39

Multiply the number in the square root answer by 20 to get the trial divisor.

$$\begin{array}{r} 2 \\ \sqrt{\overline{6}\,\overline{36.}} \\ \underline{4} \\ 2 \quad 36 \end{array} \leftarrow \text{1st dividend}$$

20 x ____ = ____

Answer to frame 7-54

Correct response to frame 7-54 is in panel 7-2, page 7-36.

FRAME 7-55

The square root of fractions can also be extracted. When the numerator and denominator are perfect squares, finding the square root is relatively simple. You merely take the square root of both the numerator and demonination. Examples:

$$\sqrt{\frac{4}{9}} = \frac{2}{3}\ ; \qquad \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Find the square root of the fractions below.

a. $\sqrt{\frac{4}{25}}$ b. $\sqrt{\frac{64}{81}}$

c. $\sqrt{\frac{9}{25}}$

SET 2. ELEMENTS OF A POWER

(A power is a product obtained by using a base a certain number of times as a factor) (7-5)

FRAME 7-6

A short, concise method of indicating the multiplication operations of raising a number to a power is used. Using this method, 2 x 2 x 2 x 2 can be written as 2^4. The small figure, placed at the right and above the base shows how many times the base is used as a factor. This small figure is called an EXPONENT. In the expression 2^4, the ^{4} is the _____ and the 2 is the _____.

(square root) (7-22)

FRAME 7-23

Select, from the statements below, the statement that defines the square root of a number. Circle the letter in front of your answer.

a. The square root of a number is one of two factors of a number.

b. The square root of a number is one of the two equal factors of a number.

c. The product of two numbers, when multiplied together, is the square root of a number.

Answer to frame 7-39

You should have this:

$$\begin{array}{r} 2\ . \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \end{array}$$

20x2 = 40

FRAME 7-40

Estimate the number of times the trial divisor will divide into the dividend and write the number estimated into the square root directly above the pair of digits in the radicand that you brought down as part of the dividend.

$$\begin{array}{r} 2\ . \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \end{array}$$

1st trial divisor 20x2 = 40 — 1st dividend

Answer to frame 7-55

a. $\frac{2}{5}$

b. $\frac{8}{9}$

c. $\frac{3}{5}$

FRAME 7-56

When a fraction is <u>NOT</u> a perfect square, finding the square root becomes more involved. You change the fraction to a decimal and proceed as with any square root problem involving decimals. Example: $\sqrt{\frac{4}{8}}$, changing $\frac{4}{8}$ to a decimal $\begin{array}{r} .5 \\ 8\overline{)4.0} \end{array}$ and $\begin{array}{r} .\ 7\ \ 0\ \ 7 \\ \sqrt{\widehat{.50}\ \widehat{00}\ \widehat{00}} \end{array}$ + remainder.

Change the fraction below to a decimal and arrange it for extraction of the square root. Do not extract the square root.

$$\frac{3}{5}$$

(exponent; base) (7-6)

FRAME 7-7

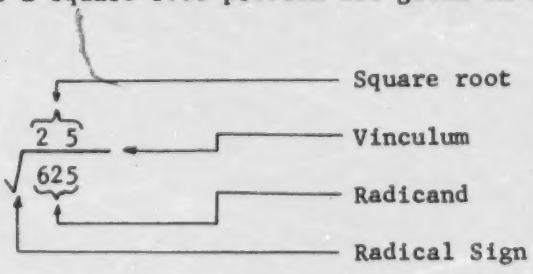
Using the exponent method of indicating that a number is to be raised to a power, 5 x 5 x 5 can be written 5^3. In this expression, the 5 is the _____ and the ^{3} is the _____.

SET 8. SOLVING FOR THE SQUARE ROOT OF A NUMBER

(b. The square root of a number is one of the two equal factors of a number) (7-23)

FRAME 7-24

The four parts of a square root problem are given in the example below.



The number above the vinculum is the _____ _____ of the radicand.

Answer to frame 7-40

You should have this:

$$\begin{array}{r} 2\ \ 5. \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \end{array}$$

$20x2 = 40$

FRAME 7-41

Add the estimation to the trial divisor and multiply that sum by the estimation to get the product.

$$\begin{array}{r} 2\ \ . \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \end{array}$$

$20x2 = 40$

Answer to frame 7-56

$$\sqrt{.\widehat{60}\ \widehat{00}\ \widehat{00}}$$

FRAME 7-57

Find the square root of the numbers below. When the answer contains decimals, carry to three decimal places.

a. $\sqrt{\frac{25}{64}}$

b. $\sqrt{\frac{4}{5}}$

SET 3. SQUARE OF A NUMBER

(base; exponent) (7-7)

FRAME 7-8

Anytime a number is multiplied by itself, it is said to be squared. The square is the most commonly used power. The square of a number is that number multiplied by _____.

(square root) (7-24)

FRAME 7-25

A method used to arrange a number for extraction of its square root is shown below.

First, place the radical sign and vinculum around the radicand: $\sqrt{4}$ and $\sqrt{25}$. Next, place a decimal in the radicand (if one does not already exist): $\sqrt{4.}$ and $\sqrt{25.}$; THEN place a decimal above the vinculum, directly over the decimal in the radicand: $\sqrt{4.}$ or $\sqrt{25.}$. Finally arrange the digits in groups of two from the decimal point $\sqrt{4.}$ or $\sqrt{25.}$. When the number is larger, the digits will be marked off like this: $\sqrt{225.}$ and $\sqrt{4225.}$.

Arrange the numbers below for extraction of the square root.

327 and 6632

Answer to frame 7-41

You should have this:

$$\begin{array}{r} 2\ 5. \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \end{array}$$

$$\begin{array}{r} 20 \times 2 = 40 \\ +\ 5 \\ \hline 45 \\ \times\ 5 \\ \hline 225 \end{array}$$

FRAME 7-42

Place the product obtained in the preceding frame beneath the dividend and subtract. If the product is larger than the dividend, reduce the estimate and repeat the step given in the preceding frame.

$$\begin{array}{r} 2\ 5. \\ \sqrt{6\ 36.} \\ 4 \\ \hline 2\ 36 \end{array} \qquad \begin{array}{r} 20 \times 2 = 40 \\ +\ 5 \\ \hline 45 \\ \times\ 5 \\ \hline 225 \end{array}$$

_____ ← Remainder

SET 9. CHECKING A SQUARE ROOT PROBLEM FOR ACCURACY

Answer to frame 7-57

a. $\frac{5}{8}$

b. .894 + remainder

FRAME 7-58

To check that you have extracted the square root of a perfect square, SQUARE the square root. Example:

$$\begin{array}{r} 2\ 5. \\ \sqrt{\widehat{6}\ \widehat{25}.} \\ \hline \end{array} \quad = \quad \begin{array}{r} 25 \\ \times\ 25 \\ \hline 625 \end{array}$$

To determine if your work is correct, when extracting the square root of a perfect square, you _____ your answer.

(itself) (7-8)

FRAME 7-9

When a number is multiplied by itself, it is said to be

_____.

$\sqrt{\widehat{327}.}$; $\sqrt{\widehat{6632}.}$ (7-25)

FRAME 7-26

As you can see from the answers to the previous frame, the arrangement to the extreme left of the decimal does not always contain two digits. This does not introduce any problems in the determination of the square root of the number. You start at the decimal and work to the left, grouping the digits in pairs until all possible pairs are marked off.

When the number below is properly arranged for extraction of its square root, which digit will <u>not</u> be part of a pair?

$\sqrt{63257.}$ Answer _____.

Answer to frame 7-42

You should have this:

$$\begin{array}{r} 2\ \ 5. \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 20x2 = 40 \\ +\ \ 5 \\ \hline 45 \\ \times\ \ 5 \\ \hline 225 \end{array}$$

Remainder

FRAME 7-43

When you have a remainder in square root and desire to carry the problem out to a number of decimal places for greater accuracy, add pairs of zeros to the radicand. Unless otherwise indicated, carry the square root to at least three decimal places.

To continue the problem below, add a pair of zeros to the radicand on the right of the decimal. Then bring that pair of zeros down beside the remainder, (11). This gives you the 2nd dividend.

$$\begin{array}{r} 2\ \ 5. \\ \sqrt{\widehat{6}\ \widehat{36}.} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 20 \times 2 = 40 \\ +\ \ 5 \\ \hline 45 \\ \times\ \ 5 \\ \hline 225 \end{array}$$

(square) (7-58)

FRAME 7-59

When extracting the square root from a number that is <u>not</u> a perfect square, there will be a remainder. To check your work, square your answer and then add the remainder.

To check your work when extracting the square root of a number that is not a perfect square, first square the square root and then _____ the _____.

(squared) (7-9)

FRAME 7-10

When a number is to be squared, you can multiply the number by itself, or express it by the exponent ^{2}.

If you were told to square 6, you could multiply 6 x 6 = 36, but if you expressed it with the exponent ^{2}, it would look like this: 6^{2}.

Then if you were to square 7, you would get _____. Also, to express 7 squared by using an exponent, it would be written _____.

---

(6) (7-26)

FRAME 7-27

When you are to extract the square root of a number that has digits to the right of the decimal, you must work with pairs, even if it becomes necessary to add a zero.

For example: Place the decimal and arrange .75 as $\sqrt{.75}$; .7542 as: $\sqrt{.7542}$ and .625 as $\sqrt{.6250}$

Now arrange this number for extraction of its square root.

.72646

Answer to frame 7-43

You should have this:

```
          2  5.
       √6 36.00
          4
20x2 = 40 2 36
     +  5 2 25
       45   11 00
     x  5
      225
```

2nd dividend

FRAME 7-44

Multiply 20 by the number in the square root answer. This gives you the 2nd trial divisor.

```
          2  5.
       √6 36.00
          4
20 x 2 = 40 2 36
     +  5 2 25
       45   11 00   2nd dividend
     x  5
      225
```

20 x __ = ____

(add; remainder) (7-59)

FRAME 7-60

Before the remainder can be added to the square of the square root, the decimal must be brought down from the radicand and placed in the remainder. The remainder will have as many decimal places as the radicand, even if this requires placing zeros between the decimal and the other digits. See example below.

```
          2  5. 2  1  9
       √6 36.00 00 00
          4
          2 36
          2 25
            11 00
            10 04
               96 00
               50 41
               45 59 00
               45 38 61
                .00 20 39
```

Decimal placed in remainder

Accuracy Proven

```
          25.219
        x 25.219
          226971
           25219
           50438
          126095
           50438
        635.997961
        + .002039
        636.000000
```

Remainder added

If the radicand has eight decimal places, the remainder will have _____ decimal places.

(49; 7^2) (7-10)

FRAME 7-11

Besides whole numbers, decimals, fractions and even letters can be squared. Examples:

1. To square a decimal like .9, you would compute .9 x .9 = .81
2. To square a fraction like $\frac{3}{4}$, you would compute $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$.
3. To square a letter like R, you would compute R x R = R^2. Notice, since you do not know the value of R, you must express the square in the exponent form.

Now, the square of .08 is _____, and the square of $\frac{2}{3}$ is _____.

$\left[\sqrt{.\widehat{72}\widehat{64}\widehat{60}}\right]$ (7-27)

FRAME 7-28

Sometimes the number from which you are to extract the square root extends on both sides of the decimal. In such a case, you merely apply the rules for arrangement, (marking off the radicand into pairs) first to the left side of the decimal and then to the right side. Example: You arrange 632.745 as:

$$\sqrt{\widehat{63}\widehat{2}.\widehat{74}\widehat{50}}$$

Arrange the number below for extraction of its square root.

85134.523

Answer to
frame 7-44

You should have this:

$$\begin{array}{r} 2\ 5. \\ \sqrt{6\ 36.00} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \end{array}$$

20x2 = 40
+ 5
45
x 5
225

20 x 25 = 500
2nd trial divisor

FRAME 7-45

Estimate the number of times the 2nd trial divisor will divide into the 2nd dividend. Write the estimation into the square root directly above the pair of digits in the radicand that you brought down to form part of the 2nd dividend.

NOTE: We will no longer show the work that was done here to save space on this page.

$$\begin{array}{r} 2\ 5. \\ \sqrt{6\ 36.00} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \end{array}$$

20 x 25 = 500
2nd trial divisor

(eight) (7-60)

FRAME 7-61

Extract the square root of the number below and prove your work for accuracy. Show all work below. Work to three decimal places.

$$\sqrt{5642.0}$$

$\left[.0064;\ \frac{4}{9}\right]$ (7-11)

FRAME 7-12

Square the numbers listed below. Place your answer in front of the letter.

_____ a. 3

_____ b. 12

_____ c. .6

_____ d. $\frac{1}{2}$

_____ e. 1.5

_____ f. $\frac{4}{9}$

$\left[\sqrt{8\ \widehat{51}\ \widehat{34}.\widehat{52}\ \widehat{30}}\right]$ (7-28)

FRAME 7-29

After arranging, the next step in extracting the square root of a number is to find a number that is the largest perfect square contained in the digit or pair of digits furthest to the left in the radicand.

NOTE: The next few frames will show the steps needed to find the square root of the number below.

$$\sqrt{\widehat{5}\ \widehat{29}.}$$

The largest perfect square in 5 is 4.

Place the perfect square, four (4), below the first digit in the radicand. Then place its square root, two (2), directly above the digit in the radicand. (Go ahead and write it in the problem below)

$$\sqrt{\widehat{5}\ \widehat{29}.}$$

Answer to
frame 7-45

You should have
this:

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{\widehat{6}\ \widehat{36.00}} \\ \underline{4} \\ 2\ 36 \\ \underline{2\ 25} \\ 11\ 00 \end{array}$$

20 x 25 = 500

FRAME 7-46

Add your estimation to the 2nd trial divisor and multiply that sum by the estimation.

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{\widehat{6}\ \widehat{36.00}} \\ \underline{4} \\ 2\ 36 \\ \underline{2\ 25} \\ 11\ 00 \end{array}$$

20 x 25 = 500

SET 10. ROOT OF PERFECT POWERS

(Correct response to frame 7-61 is on page 7-37, panel 7-3)

FRAME 7-62

Roots other than the square root of a number can be determined. Anyone of three equal factors of a number is called the CUBE ROOT of that number. For example, 2 is the cube root of 8.

The cube root of 27 is _____, which is one of three equal factors you can _____together to give you 27.

[a. 9; b. 144; c. .36; d. $\frac{1}{4}$; e. 2.25; f. $\frac{16}{81}$] (7-12)

FRAME 7-13

From the statements below, select the one that describes the square of a number. Circle the letter in front of your answer.

a. The square of a number is that number multiplied by 2.

b. Any number multiplied by an even number is squared.

c. The square of a number is that number multiplied by itself.

Answer to frame 7-29

You should have this:

$$\begin{array}{r} 2\ . \\ \sqrt{5\ 29.} \\ \underline{4} \end{array}$$

FRAME 7-30

Now subtract the perfect square, 4, from 5 in the radicand, and bring the next pair of digits in the radicand down beside the remainder. This will give you the first dividend--129.

$$\begin{array}{r} 2\ . \\ \sqrt{5\ 29.} \\ \underline{4} \end{array}$$

Answer to
frame 7-46

You should have this:

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{6\ \widehat{36.00}} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \end{array}$$

20 x 25 = 500
+ 2
502
x 2
1004

FRAME 7-47

Place the product obtained in the preceding frame beneath the 2nd dividend and subtract. (Remember, if the product is larger than the dividend, reduce the estimate and repeat the step given in the preceding frame).

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{6\ \widehat{36.00}} \\ 4 \\ \hline 2\ 36 \\ 2\ 25 \\ \hline 11\ 00 \end{array}$$

20 x 25 = 500
+ 2
502
x 2
1004

(3; multiply) (7-62)

FRAME 7-63

The symbol that has been used to indicate the square root of a number is $\sqrt{}$. The symbol used to indicate the cube root is $\sqrt[3]{}$. The small number at the upper left corner of the radical sign is called the INDEX. The index shows what root is to be determined. $\sqrt[5]{}$ is the sign for the _____ root.

SET 4. PERFECT SQUARE

(c. The square of a number is that number multiplied by itself)(7-13)

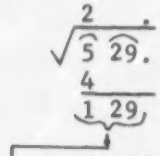
FRAME 7-14

A PERFECT square is a number that is the exact square of another number.

An example of a perfect square is 81. It is the exact square of 9; that is, 9 x 9 = 81.

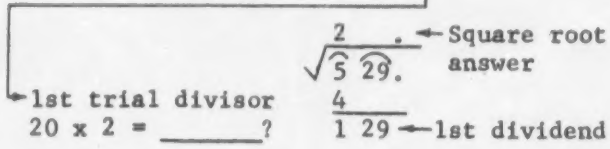
When a number is the exact square of another number, that number is said to be a _____ _____.

Your answer to frame 7-30 should resemble this:



FRAME 7-31

You now have to get a trial divisor. To do this, multiply 20 by the number in the square root answer. What is the trial divisor for the problem below?



Answer to frame 7-47

You should have this:

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{\widehat{6}\ \widehat{36}.\widehat{00}} \\ \underline{4} \\ 2\ 36 \\ \underline{2\ 25} \\ 11\ 00 \\ \underline{10\ 04} \\ 96 \end{array}$$

$$\begin{array}{r} 500 \\ +\ 2 \\ \hline 502 \\ \times\ 2 \\ \hline 1004 \end{array}$$

Remainder

FRAME 7-48

Place another pair of zeros into the radicand and bring that pair of zeros down beside the remainder. This gives you the 3rd dividend.

$$\begin{array}{r} 2\ \ 5.\ 2 \\ \sqrt{\widehat{6}\ \widehat{36}.\widehat{00}} \\ \underline{4} \\ 2\ 36 \\ \underline{2\ 25} \\ 11\ 00 \\ \underline{10\ 04} \\ 96 \end{array}$$

$$\begin{array}{r} 20 \times 25 = 500 \\ +\ 2 \\ \hline 502 \\ \times\ 2 \\ \hline 1004 \end{array}$$

(fifth) (7-63)

FRAME 7-64

If no index is shown, it is understood to be 2 or the square root. Determine the index for the following problems.

a. $\sqrt{36}$ b. $\sqrt[3]{8}$

c. $\sqrt[4]{81}$ d. $\sqrt[7]{128}$

a. _____ b. _____ c. _____ d. _____

(perfect square) (7-14)

FRAME 7-15

The whole numbers between 1 and 100 that are perfect squares are: 1 - 4 - 9 - 16 - 25 - 36 - 49 - 64 - 81 and 100.

For example: The number 64 is the square of 8. Eight (8) is one of two equal factors (numbers) which you multiply together to get 64. 8 x 8 = 64, making 64 a perfect square.

Would 10 be a perfect square?

Answer (yes/no)

---

(40) (7-31)

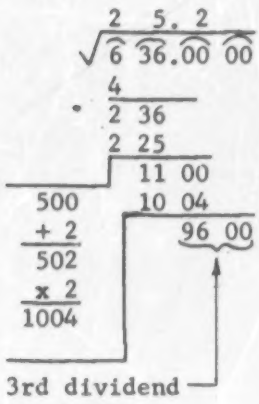
FRAME 7-32

The number 20 is a constant that is multiplied by the number in the square root answer to obtain a trial divisor.

Suppose you have a number in the square root answer of 13.1 and you must take the answer out one more decimal place. To get a trial divisor, you multiply _____ x 13.1.

Answer to
frame 7-48

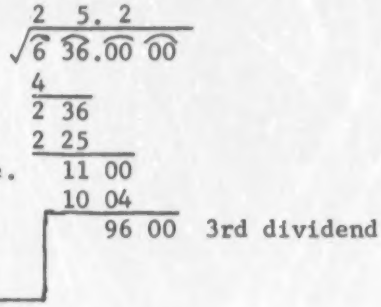
You should have



FRAME 7-49

Multiply 20 by the number in the square root answer. This gives you the 3rd trial divisor.

NOTE: We will no longer show the work that was done here to save space on this page.



3rd trial divisor
20 x 252 = _____

NOTE: Multiply 20 x 252 instead of 25.2. Since the decimal is already placed in the square root, we can disregard it and work as if all the numbers are whole numbers.

(a. 2; b. 3; c. 4; d. 2) (7-64)

FRAME 7-65

There is no simple mathematical procedure for determining roots other than the square root. Roots of higher power can often be determined for perfect power by recognizing that the number is a certain perfect power. For example, in $\sqrt[3]{125}$, it can be recognized that the cube root is 5 because 5 x 5 x 5 = 125.

$\sqrt[4]{16}$ is _____

(No, because there are NO two equal factors that can be multiplied together to get 10.) (7-15)

FRAME 7-16

From the numbers listed below, select the perfect squares. Place a PS in front of your choices.

a. 16	d. 12	g. 99
b. 3	e. 100	h. 1
c. 25	f. 64	

---

(20) (7-32)

FRAME 7-33

When you get the trial divisor, you estimate the number of times the trial divisor will divide into the dividend. You then write the estimation into the square root answer directly above the pair of digits in the radicand that you brought down to form part of the dividend.

In the problem below, estimate how many times the 1st trial divisor will go into the 1st dividend and write the estimation in its proper place.

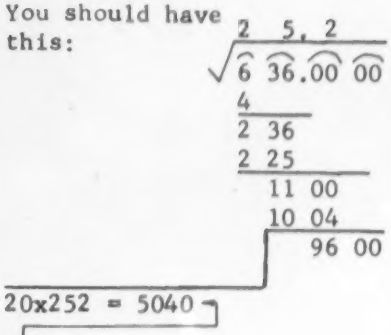
$$\begin{array}{r} 2 \\ \hline \sqrt{5\ 29.} \\ 4 \\ \hline 1\ 29 \end{array}$$

1st trial divisor
$20 \times 2 = 40$

1st dividend

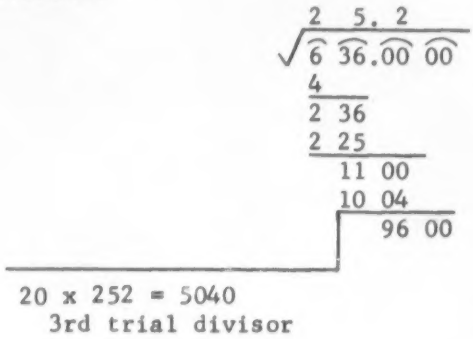
Answer to
frame 7-49

You should have this:



FRAME 7-50

Estimate the number of times the 3rd trial divisor will divide into the 3rd dividend. Write the estimation into the square root directly above the pair of digits in the radicand that you brought down to form part of the 3rd dividend.



Turn back to the bottom of page 7-3.

---

(2) (7-65)

FRAME 7-66

You have completed the instructional portion of this program. You may review the objectives and content of the program if you wish.

A SELF-TEST begins on page 7-38.

SET 5. SQUARING NUMBERS

(Perfect Squares: a.; c.; e.; f.; h.) (7-16)

FRAME 7-17

Write the square of the numbers listed below. Place the square of each number in front of the letter.

_____ a. 10

_____ b. $\frac{2}{3}$

_____ c. 8

_____ d. 1.4

_____ e. $\frac{3}{4}$

_____ f. 5

_____ g. .12

_____ h. .4

Turn back to the bottom of page 7-2.

Answer to frame 7-33

Your estimation should be 3.

The problem should now resemble this:

$$\begin{array}{r} 2\ \ 3. \\ \sqrt{5\ 29.} \\ 4 \\ \hline 1\ 29 \end{array}$$

20x2 = 40

FRAME 7-34

Now, add the estimation (3) to the trial divisor (40) and multiply that sum by the estimation (3). If the product of these numbers is larger than the dividend, reduce the estimation and repeat the procedure above. Find the sum of divisor and estimation and product of the sum and estimation in the problem below.

$$\begin{array}{r} 2\ \ 3. \\ \sqrt{5\ 29.} \\ 4 \\ \hline 1\ 29 \end{array}$$

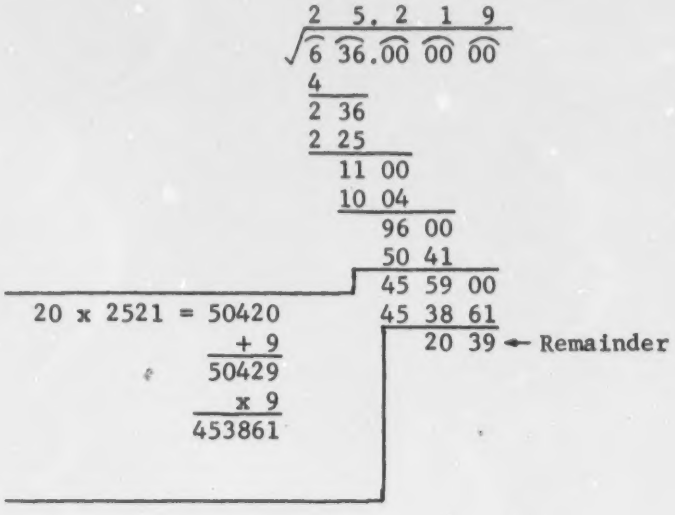
1st trial divisor — 1st dividend

20 x 2 = 40
Estimate + 3
Sum →
Estimate x 3
Product →

Turn back to the top of page 7-3.

PANEL 7-1

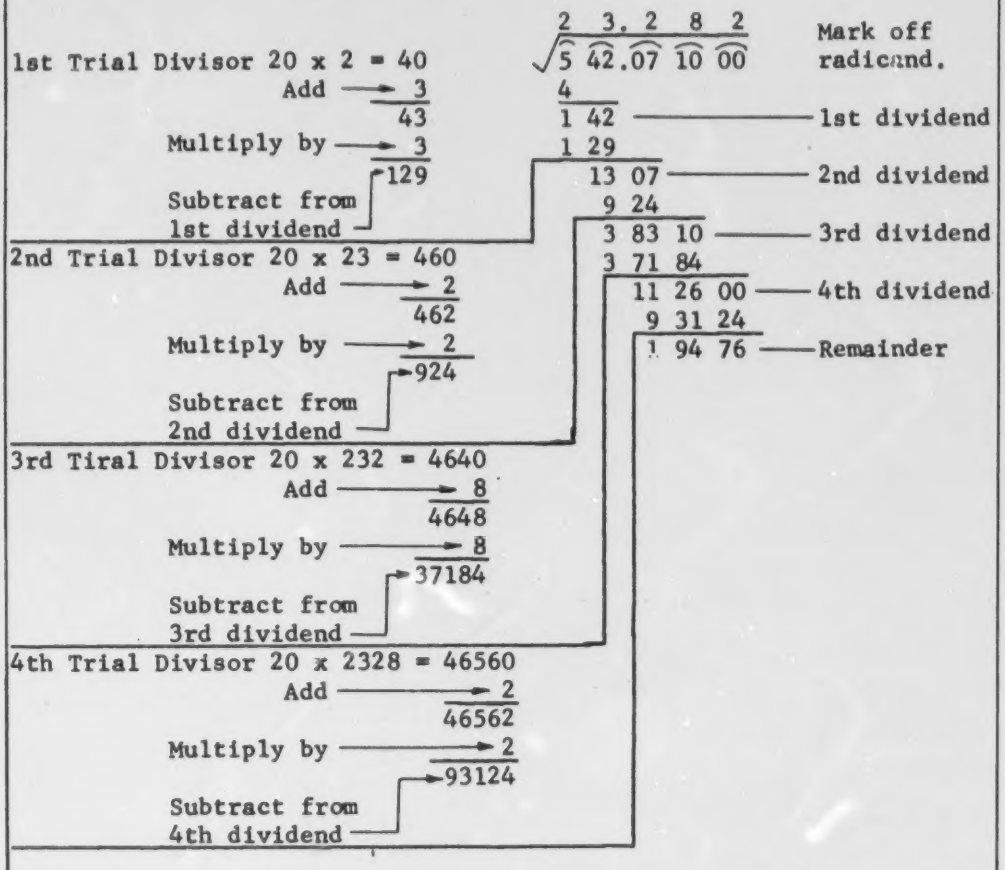
Your problem should now be like this:



NOTE: Go to page 7-9, frame 7-54.

PANEL 7-2

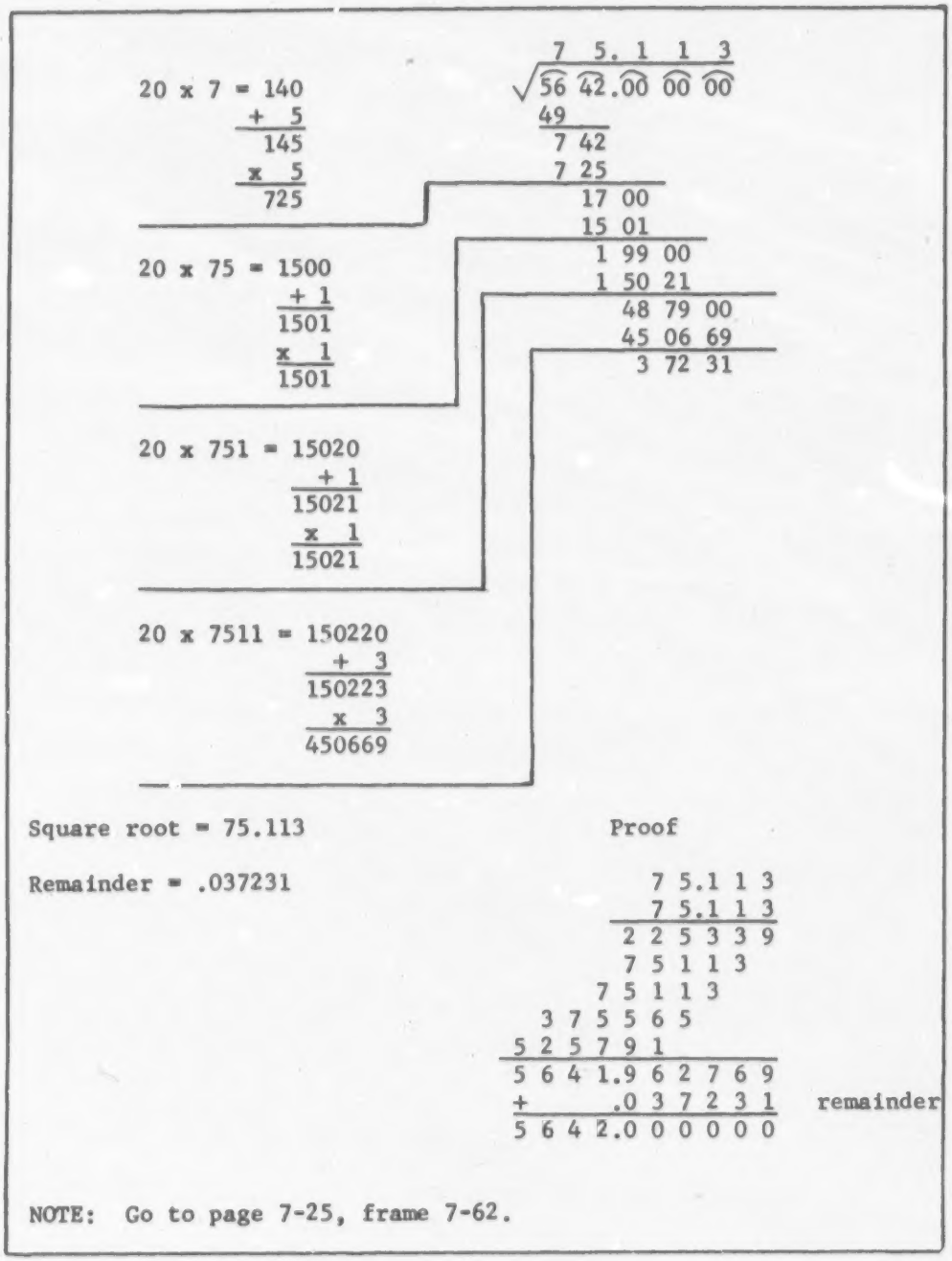
Answer and proper procedure for frame 7-54, page 7-9.



If you had trouble using correct procedure or finding the correct square root, return to page 7-5, frame 7-36 and review square root.

If your square root and procedure are correct, turn to page 7-11, frame 7-55 and proceed with the lesson.

PANEL 7-3



Square root = 75.113

Remainder = .037231

Proof

```
      7 5.1 1 3
      7 5.1 1 3
    2 2 5 3 3 9
    7 5 1 1 3
  7 5 1 1 3
3 7 5 5 6 5
5 2 5 7 9 1
5 6 4 1.9 6 2 7 6 9
+       .0 3 7 2 3 1   remainder
5 6 4 2.0 0 0 0 0 0 0
```

NOTE: Go to page 7-25, frame 7-62.

SELF-TEST

POWERS AND ROOTS

1. Write in your own words the definition of a power of a number.

_____

_____.

2. In the expression, 3^4, write the names of the elements used to indicate the arithmetic operation of raising a number to a power.

3 _____

4 _____

3. Write in your own words the definition of the square of a number.

_____

_____.

4. From the numbers listed below, select the perfect squares.

Place a PS in front of the perfect squares.

_____ a. 24	_____ e. 4	_____ i. 100
_____ b. 16	_____ f. 65	_____ j. 36
_____ c. 34	_____ g. 49	_____ k. 99
_____ d. 1	_____ h. 86	_____ l. 25

5. Write the square of the numbers listed below. Place the square of each number in front of the letter.

_____ a. 4	_____ e. 3	_____ i. 20
_____ b. 12	_____ f. 15	_____ j. 5
_____ c. 1	_____ g. 9	_____ k. .5
_____ d. 8	_____ h. 7	_____ l. $\frac{1}{2}$

6. Write the names of the higher power of the number listed below.

5^4 is the _____ power of 5

3^3 is the _____ power of 3

7^5 is the _____ power of 7

7. Write in your own words the definition of the square root of a number.

_____

_____

8. Solve the problems below by extracting the square root of each whole number.

a. $\sqrt{225}$ b. $\sqrt{4264}$

9. Solve the problems below by extracting square root of each fraction and decimal.

a. $\sqrt{.09}$ b. $\sqrt{\frac{9}{16}}$

c. $\sqrt{.4}$ d. $\sqrt{\frac{3}{5}}$

e. $\sqrt{5.32}$

10. Prove your answer to problem 9e above.

11. Solve the problems below by determining the root of each of the perfect powers.

a. $\sqrt[3]{27}$

b. $\sqrt[4]{1296}$

ANSWERS TO SELF-TEST LESSON 7

1. A power is a product obtained by using a base a certain number of times as a factor.

2. 3 - <u>base</u>

   • - <u>exponent</u>

3. The square of a number is that number multiplied by itself.

4. b. $16 = 4 \times 4$ i. $100 = 10 \times 10$

   d. $1 = 1 \times 1$ j. $36 = 6 \times 6$

   e. $4 = 2 \times 2$ l. $25 = 5 \times 5$

   g. $49 = 7 \times 7$

5. 16 a. $4 \times 4 = 16$ 81 g. $9 \times 9 = 81$

   144 b. $12 \times 12 = 144$ 49 h. $7 \times 7 = 49$

   1 c. $1 \times 1 = 1$ 400 i. $20 \times 20 = 400$

   64 d. $8 \times 8 = 64$ 25 j. $5 \times 5 = 25$

   9 e. $3 \times 3 = 9$ .25 k. $.5 \times .5 = .25$

   225 f. $15 \times 15 = 225$ $\frac{1}{4}$ l. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

6. 5^4 is the <u>fourth</u> power of 5.

   3^3 is the <u>third</u> power of 3.

   7^5 is the <u>fifth</u> power of 7.

7. The square root of a number is one of the two equal factors of a number, which, when multiplied together, give us the number.

8. a. Square root 15
remainder 0

b. Square root 65.299
remainder 0.040599

$$\begin{array}{r} 1\ 5. \\ \sqrt{\widehat{2}\ \widehat{25}.} \\ 1 \\ \hline 1\ 25 \\ 1\ 25 \\ \hline 0 \end{array}$$

20x1 = 20
+ 5
25
x 5
125

$$\begin{array}{r} 6\ 5.\ 2\ 9\ 9 \\ \sqrt{\widehat{42}\ \widehat{64}.\widehat{00}\ \widehat{00}\ \widehat{00}} \\ 36 \\ \hline 6\ 64 \\ 6\ 25 \\ \hline 39\ 00 \\ 26\ 04 \\ \hline 12\ 96\ 00 \\ 11\ 74\ 41 \\ \hline 1\ 21\ 59\ 00 \\ 1\ 17\ 53\ 01 \\ \hline 4\ 05\ 99 \end{array}$$

20x6 = 120
+ 5
125
x 5
625

20x65 = 1300
+ 2
1302
x 2
2604

20x652 = 13040
+ 9
13049
x 9
117441

20x6529 = 130580
+ 9
130589
x 9
1175301

9. a. Square root - 0.3
remainder - 0

$$\begin{array}{r} .\ 3 \\ \sqrt{.\widehat{09}} \\ .09 \\ \hline 0 \end{array}$$

b. Square root - $\frac{3}{4}$

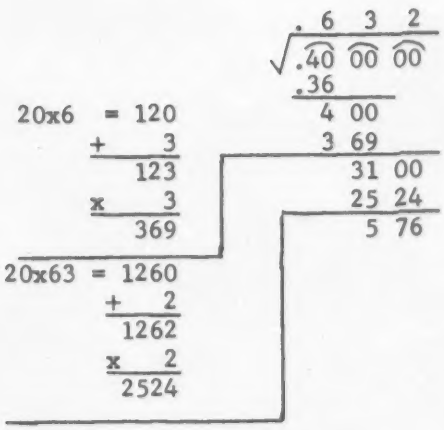
remainder - 0

$$\sqrt{\frac{9}{16}}$$

Perfect Square

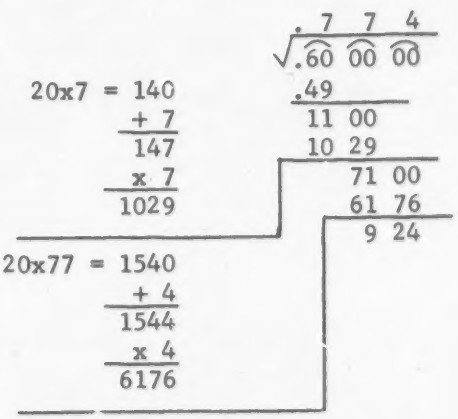
$$\frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

c. Square root - .632
remainder - .000576

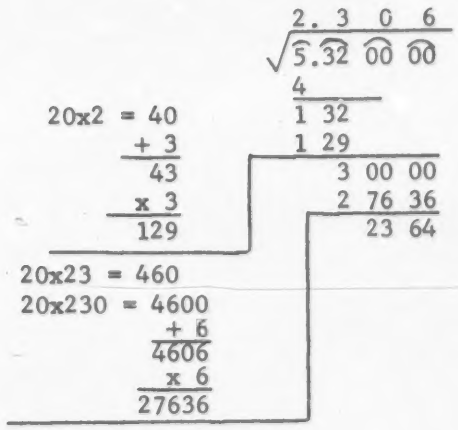


d. Square root - .774
remainder - .000924

$$\sqrt{\frac{3}{5}} = \sqrt{.6}$$



e. Square root - 2.306
remainder - .002364



7-43

10.

$$
\begin{array}{r}
2.306 \\
\underline{2.306} \\
13836 \\
69180 \\
\underline{4612} \\
5.317636 \\
\underline{+.002364} \\
5.320000
\end{array}
$$

11. a. $\sqrt[3]{27} = 3$ $\qquad 3 \times 3 \times 3 = 27$

b. $\sqrt[4]{1296} = 6$ $\qquad 6 \times 6 \times 6 \times 6 = 1296$

* U. S. GOVERNMENT PRINTING OFFICE : 1974 732-078/183